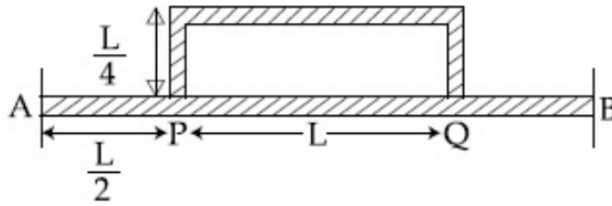# Physics

Single Correct Questions

1. Temperature difference of $120^\circ C$ is maintained between two ends of a uniform rod AB of length $2L$. Another bent rod PQ, of same corss-section as AB and length $\dfrac{3L}{2}$, is connected across AB (See figure). In steady state, temperature difference between P and Q will be close to :


(A) $75^\circ C$ (B) $60^\circ C$ (C) $45^\circ C$ (D) $35^\circ C$

2. A mixture of 2 moles of helium gas (atomic mass = 4u), and 1 mole of argon gas (atomic mass = 40 u) is kept at 300 K in a container. The ratio of their rms speeds $\left[\dfrac{V_{rms}(helium)}{V_{rms}(argon)}\right]$, is close to :

(A) 0.32   (B) 0.45   (C) 3.16   (D) 2.24

3. An L-shaped object, made of thin rods of uniforms mass density, is suspended with a string as shown in figure. If AB=BC, and the angle made by AB with downward vertical is θ, then :
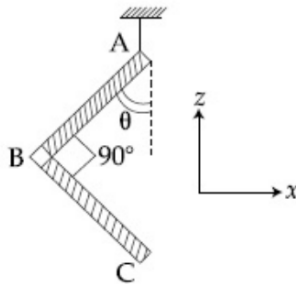

(A) $\tan\theta = \dfrac{1}{2\sqrt{3}}$ (B) $\tan\theta = \dfrac{1}{2}$ (C) $\tan\theta = \dfrac{1}{\sqrt{3}}$ (D) $\tan\theta = \dfrac{1}{3}$

4. A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is :

(A) 2.5%   (B) 1.0%   (C) 0.5%   (D) 2.0%

5. A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion $\alpha/^\circ C$. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by $\Delta T K$. Young's modulus, Y, for this metal is :

(A) $\dfrac{F}{A\alpha(\Delta T - 273)}$ (B) $\dfrac{F}{A\alpha\Delta T}$ (C) $\dfrac{2F}{A\alpha\Delta T}$ (D) $\dfrac{2F}{A\alpha\Delta T}$

6. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass $m(m << M)$. When the car is at rest, the speed of transverse waves in the string is $60 ms^{-1}$. When the car has acceleration a, the wave-speed increases to $60.5 ms^{-1}$. The value of a, in terms of gravitational acceleration g, is closest to :

(A) $\dfrac{g}{30}$ (B) $\dfrac{g}{20}$ (C) $\dfrac{g}{10}$ (D) $\dfrac{g}{5}$

Space for rough use

7. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is:

(A) $\dfrac{R}{\sqrt{2}}$
(B) $R\sqrt{2}$
(C) $\dfrac{R}{\sqrt{5}}$
(D) R

8. Three charges $+Q, q, +Q$ are placed respectively, at distance, $0, d/2$ and d from the origin, on the x-axis. If the net force experienced by $+Q$, placed at $x = 0$, is zero, the value of q is :

(A) $+Q/4$
(B) $-Q/2$
(C) $-Q/4$
(D) $+Q/2$

9. Consider a tank made of glass (refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass inerface is never completely polarized. For this to happen, the minimum value of μ is :
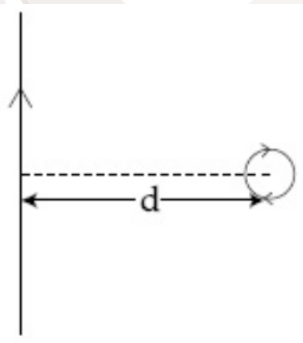

n = 1.5
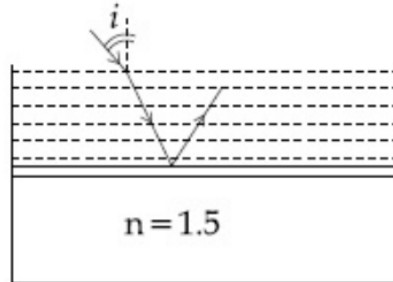
(A) $\dfrac{5}{\sqrt{3}}$
(B) $\dfrac{3}{\sqrt{5}}$
(C) $\dfrac{4}{3}$
(D) $\sqrt{\dfrac{5}{3}}$

10. A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is :

(A) $y^2 = x^2 + \text{constant}$
(B) $y = x^2 + \text{constant}$
(C) $xy\ \text{constant}$
(D) $y^2 = x + \text{constant}$

11. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3N is applied on the block. the coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block doesnot move downward?
(take $g = 10\ ms^{-2}$)
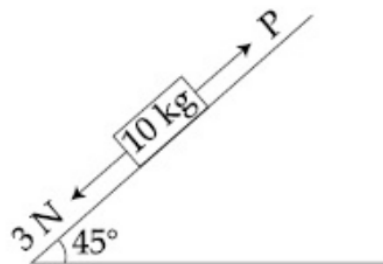
(A) 23 N
(B) 32 N
(C) 25 N
(D) 18 N

12. A parallel plate capacitor is made of two square plates of side $'a'$, separated by a distance $d(d << a)$. The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure. Capacitance of this capacitor is :
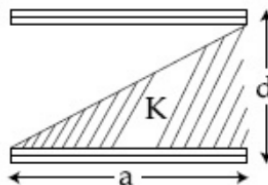
(A) $\dfrac{K \in_0 a^2}{d} \ln K$
(B) $\dfrac{1}{2} \dfrac{K \in_0 a^2}{d}$
(C) $\dfrac{K \in_0 a^2}{2d(K + 1)}$
(D) $\dfrac{K \in_0 a^2}{d(K - 1)} \ln K$

Space for rough use

13. A convex lens is put 10 cm from a light source and it makes sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screen is shifted by a distance d. Then d is :

(A) 11 cm away from the lens

(B) 0.55 cm towards the lens

(C) 0.55 cm away from the lens

(D) 0

14. A sample of radioactive material A, that has an activity of $10mCi(1\ Ci = 3.7 \times 10^{10}\ decays/s)$, has twice the number of nuclei as another sample of a different radioactive material B which has an activity of $20mCi$. The correct choice for half-lives of A and B would then be respectively :

(A) 20 days and 10 days

(B) 10 days and 40 days

(C) 20 days and 5 days

(D) 5 days and 10 days

15. A bar magnet is demagnetized by inserting it inside a solenoid of length 0.2 m, 100 turns, and carrying a current of 5.2 A. the coercivity of the bar magnet is :

(A) 1200 A/m   (B) 2600 A/m   (C) 285 A/m   (D) 520 A/m

16. Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross section $5\ mm^2$, is v. If the electron density in copper is $9 \times 10^{28}/m^3$ the value of v in mm/s is close to (Take charge of electron to be $= 1.6 \times 10^{-19}C$)

(A) 3   (B) 2   (C) 0.02   (D) 0.2

17. An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is $d(d >> a)$. If the loop applies a force F on the wire then :

(A) $F = 0$ (B) $F \propto \left(\dfrac{a}{d}\right)$ (C) $F \propto \left(\dfrac{a}{d}\right)^2$ (D) $F \propto \left(\dfrac{a^2}{d^3}\right)$

18. Surface of certain metal is first illuminated with light of wavelength $\lambda_1 = 350\ nm$ and then, by light of wavelength $\lambda_2 = 540\ nm$. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to :

(Energy of photon $= \dfrac{1240}{\lambda(in\ nm)}eV$)

(A) 2.5   (B) 1.8   (C) 5.6   (D) 1.4

19. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electron is $10^{19}m^{-3}$ and their mobility is $1.6\ m^2/(V.s)$ then the resistivity of the semiconductor (since it is an n-type semiconductor contribution of holes is ignored) is close to :

(A) $2\ \Omega m$ (B) $0.2\ \Omega m$ (C) $0.4\ \Omega m$ (D) $4\ \Omega m$

Space for rough use

20. Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}th$ of the initial kinetic energy is lost in whole process. What is value of M/m?
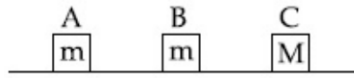
(A) 3   (B) 4   (C) 5   (D) 2

21. A current loop, having two circular arcs joined by two radial lines is shown in the figure. It carries a current of 10A. The magnetic field at point O will be close to :


i = 10 A
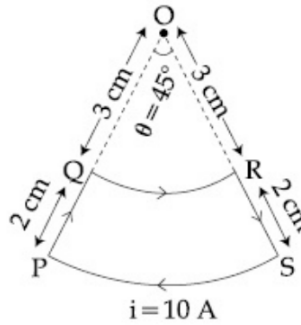
(A) $1.0 \times 10^{-7}\ T$ (B) $1.5 \times 10^{-7}\ T$ (C) $1.5 \times 10^{-5}\ T$ (D) $1.0 \times 10^{-5}\ T$

22. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is :

(A) $\dfrac{4L}{m}$ (B) $\dfrac{L}{m}$ (C) $\dfrac{L}{2m}$ (D) $\dfrac{2L}{m}$

23. A gas can be taken from A to B via two different processes ACB and ADB
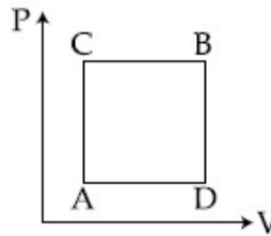
When path ACB is used 60J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10J. The heat flow into the system in path ADB is :

(A) 40 J   (B) 80 J   (C) 100 J   (D) 20 J

24. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is :
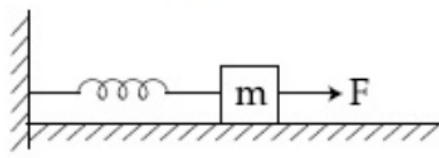
(A) $\dfrac{F}{\pi\sqrt{mk}}$ (B) $\dfrac{F}{\sqrt{mk}}$ (C) $\dfrac{2F}{\sqrt{mk}}$ (D) $\dfrac{\pi F}{\sqrt{mk}}$

Space for rough use

25. A resistance is shown in the figure. Its value and tolerance are given respectively by :

RED   ORANGE

VIOLET SILVER
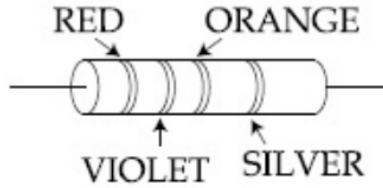
(A) $270\ \Omega, 5\%$ (B) $27\ k\Omega,\ 20\%$ (C) $270\ k\Omega,\ 10\%$ (D) $27\ k\Omega,\ 10\%$

26. A conducting circular loop made of a thin wire, has area $3.5 \times 10^{-3} m^2$ and resistance 10Ω. It is placed perpendicular to a times dependent magnetic field $B(t) = (0.4T)\sin(50\pi t)$. The field is uniform in space. Then the net charge flowing through the loop during $t = 0\ s$ and $t = 10\ ms$ is close to :

(A) 21 mC   (B) 14 mC   (C) 7 mC   (D) 6 mC

27. When the switch S, in the circuit shown, is closed, then the value of current i will be :
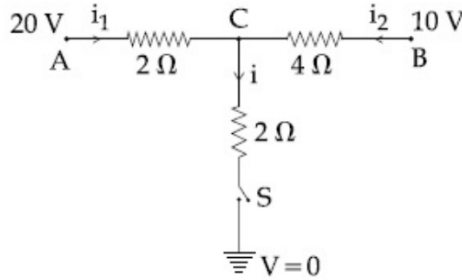
(A) 2 A   (B) 5 A   (C) 4 A   (D) 3 A

28. Two masses m and $\dfrac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Becuase of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be :
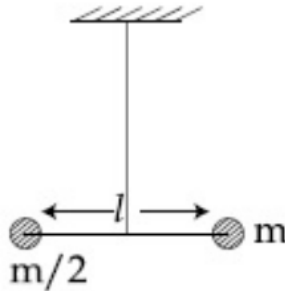
(A) $\dfrac{2k\theta_0^2}{l}$ (B) $\dfrac{k\theta_0^2}{2l}$ (C) $\dfrac{3k\theta_0^2}{l}$ (D) $\dfrac{k\theta_0^2}{l}$

29. A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive x-direction. At a particular point in space and time, $\vec{E} = 6.3\hat{j}\ V/m$. The corresponding magnetic field $\vec{B}$, at that point will be :

(A) $6.3 \times 10^{-8}\hat{k}T$ (B) $2.1 \times 10^{-8}\hat{k}T$ (C) $18.9 \times 10^{-8}\hat{k}T$ (D) $18.9 \times 10^{8}\hat{k}T$

30. Two coherent source produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio :

(A) 25 : 9   (B) 4 : 1   (C) 5 : 3   (D) 16 : 9

Space for rough use

Chemistry

1. According to molecular orbital theory, which of the following is true with respect to Li_2^+ and Li_2^-?

   (A) Li_2^+ is unstable and Li_2^- is stable
   (B) Both are stable
   (C) Both are unstable
   (D) Li_2^+ is stable and Li_2^- is unstable

2. The major product of following reaction is :
   $$R - C \equiv N \xrightarrow[(2)H_2O]{(1)AlH(i-Bu)_2} ?$$
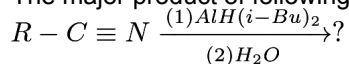
   (A) $RCONH_2$
   (B) $RCOOH$
   (C) RCH_2NH_2
   (D) $RCHO$

3. The compounds A and B in the following reaction are, respectively :

   $$\bigcirc \xrightarrow{HCHO+HCl} A \xrightarrow{AgCN} B$$
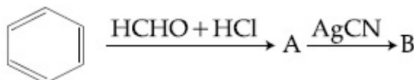
   (A) A = Benzyl alcohol, B= Benzyl isocyanide
   (B) A = Benzyl chloride, B = Benzyl isocyanide
   (C) A = Benzyl chloride, B = Benzyl cyanide
   (D) A = Benzyl alcohol, B = Bezyl cyanide

4. Consider the reversible isothermal expansion of an ideal gas in a closed system at two different temperatures T_1 and $T_2 (T_1 < T_2)$. The correct graphical depiction of the dependence of work done (w) on the final volume (V) is :
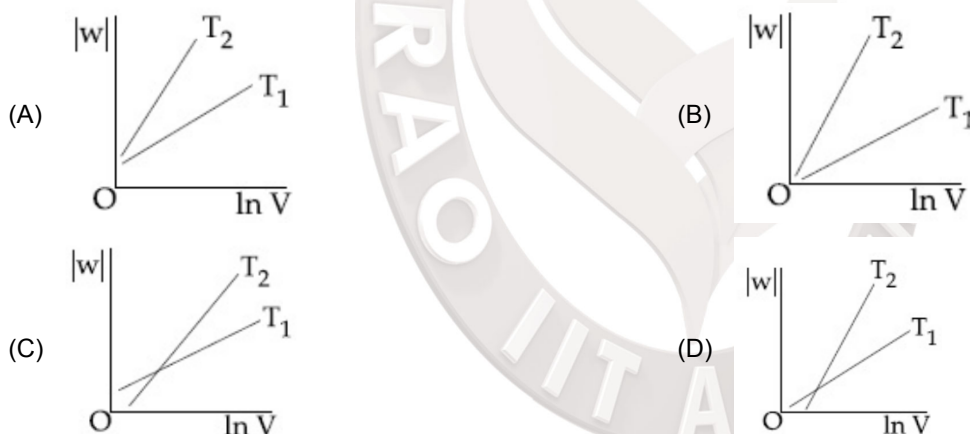
5. Adsorption of a gas follows Freundlich adsorption isotherm. In the given plot, x is the mass of the gas adsorbed on mass m of the adsorbent at pressure p. $\dfrac{x}{m}$ is proportional to :

   (A) $p^{1/2}$
   (B) p^2
   (C) p
   (D) $p^{1/4}$

Space for rough use
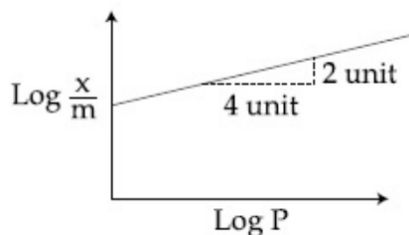
6. The following results were obtained during kinetic studies of the reaction ;
    $2A + B \rightarrow$ Products

Experiment	[A] (in mol L^{-1})	[B] (in mol L^{-1})	Initial Rate of reaction (in mol L^{-1} min^{-1})
I	0.10	0.20	6.93×10^{-3}
II	0.10	0.25	6.93×10^{-3}
III	0.20	0.30	1.386×10^{-2}

The time (in minutes) required to consume half of A is :

(A) 10   (B) 5   (C) 100   (D) 1

7. $20 \; mL$ of $0.1 \; M \; H_2SO_4$ solution is added to $30 \; mL$ of $0.2 \; M \; NH_4OH$ solution. The pH of the resultant mixture is: $[pk_b$ of $NH_4OH = 4.7]$.

(A) 5.0   (B) 9.0   (C) 9.4   (D) 5.2

8. For emission line of atomic hydrogen from $n_i = 8$ to $n_f = n,$ the plot of wave number $(\bar{v})$ against $\left(\dfrac{1}{n^2}\right)$ will be (The Rydberg constant, R_H is in wave number unit)

(A) Non linear   (B) Linear with intercept $-R_H$ (C) Linear with slope $-R_H$ (D) Linear with slope R_H

9. The ore that constains both iron and copper is :

(A) azurite   (B) dolomite   (C) copper pyrites   (D) malachite

10. Which one of the following statements regarding Henry's law is not correct?
    (A) Higher the value of K_H at a given pressure, higher is the solubility of the gas in the liquids.
    (B) The partial pressure of the gas in vapour phase is proportional to the mole fraction of the gas in the solution.
    (C) The value of K_H increases with increase of temperature and K_H is function of the nature of the gas.
    (D) Different gases have different K_H (Henry's law constant) values at the same temperature.

11. The major product of the following reaction is :
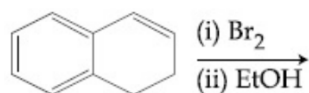
12. Aluminium is usually found in +3 oxidation state. In contrast, thallium exists in +1 and +3 oxidation states. This is due to :

(A) inert pair effect   (B) diagonal relationship   (C) lanthanoid contraction   (D) lattice effect

13. Which amongst the following is the strongest acid ?

(A) $CHCl_3$ (B) CHI_3 (C) $CHBr_3$ (D) $CH(CN)_3$

14. The anodic half-cell of lead-acid battery is recharged using electricity of 0.05 Faraday. The amount of $PbSO_4$ electrolyzed in g during the process is : (Molar mass of $PbSO_4 = 303 \; g \; mol^{-1}$)

(A) 15.2   (B) 7.6   (C) 22.8   (D) 11.4

Space for rough use

15. A water sample has ppm level concentration of the following metals :
$Fe = 0.2; Mn = 5.0; Cu = 3.0; Zn = 5.0.$ The metal that makes the water sample unsuitable for drinking is :

(A) Fe (B) Cu (C) Mn (D) Zn

16. In general, the properties that decrease and increase down a group in the periodic table, respectively, are :
(A) electronegativity and atomic radius.
(B) electron gain enthalpy and electronegativity.
(C) electronegativity and electron gain enthalpy
(D) atomic radius and electronegativity.

17. The isotopes of hydrogen are :
(A) Tritium and protium only   (B) Deuterium and tritium only
(C) Protium, deuterium and tritium   (D) Protium and deuterium only

18. The major product of the following reaction is :
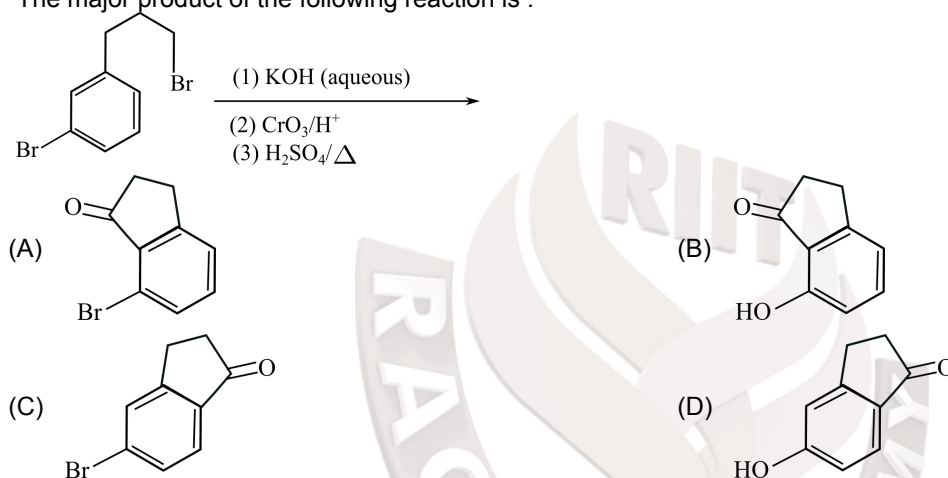
19. Major product of the following reaction is :
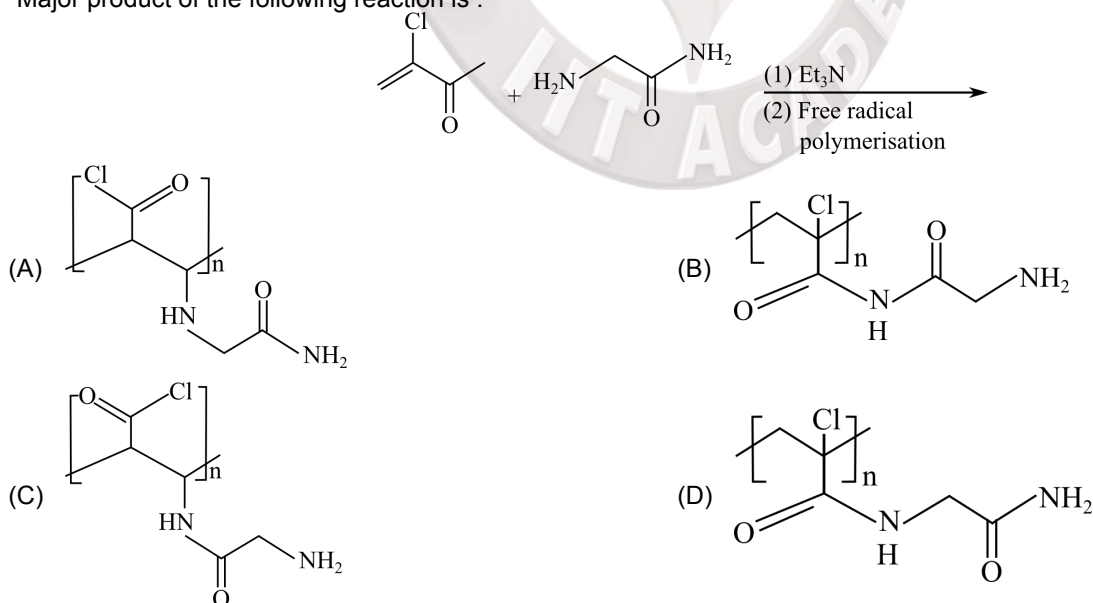
20. The correct decreasing order for acid strength is :
(A) $CNCH_2COOH > O_2NCH_2COOH > FCH_2COOH > ClCH_2COOH$
(B) $NO_2CH_2COO > FCH_2COOH > CNCH_2COOH > ClCH_2COOH$
(C) $NO_2CH_2COOH > NCCH_2COOH > FCH_2COOH > ClCH_2COOH$
(D) $FCH_2COOH > NCCH_2COOH > NO_2CH_2COOH > ClCH_2COOH$

Space for rough use

21. Correct statements among (a) to (d) regarding silicones are :
     (a) They are polymers with hydrophobic character.
     (b) They are biocompatible.
     (c) In general, they have high thermal stability and low dielectric strength.
     (d) Usually, they are resistant to oxidation and used as greases.

     (A) (a), (b) and (c) only

     (B) (a), (b) and (d) only

     (C) (a), (b), (c) and (d)

     (D) (a) and (b) only

22. A solution of sodium sulfate contains $92g$ of Na^+ ions per kilogram of water. The molality of Na^+ ions in that solution in $mol\ kg^{-1}$ is :

     (A) 4 (B) 12 (C) 16 (D) 8

23. Two complexes $[Cr(H_2O)_6]Cl_3\ (A)$ and $[Cr(NH_3)_6]Cl_3\ (B)$ are violet and yellow coloured, respectively. The incorrect statement regarding them is :

     (A) Δ_0 value for (A) is less than that of (B).

     (B) both absorb energies corresponding to their complementary colors.

     (C) Δ_0 values of (A) and (B) are calculated from the energies of violet and yellow light, respectively.

     (D) both are paramagnetic with three unpaired electrons.

24. The highest value of the calculated spin only magnetic moment (in BM) among all the transition metal complexes is :

     (A) 3.87 (B) 6.93 (C) 5.92 (D) 4.90

25. The one that is extensively used as a piezoelectric material is :

     (A) mica (B) tridymite (C) quartz (D) amorphous silica

26. The alkaline earth metal nitrate that does not crystallise with water molecules, is :

     (A) $Sr(NO_3)_2$ (B) $Ba(NO_3)_2$ (C) $Ca(NO_3)_2$ (D) $Mg(NO_3)_2$

27. 0.5 moles of gas A and x moles of gas B exert a pressure of $200Pa$ in a container of volume $10m^3$ at $1000K$. Given R is the gas contant in $JK^{-1}\ mol^{-1}, x$ is :

     (A) $\dfrac{2R}{4+R}$ (B) $\dfrac{4-R}{2R}$ (C) $\dfrac{2R}{4-R}$ (D) $\dfrac{4+R}{2R}$

28. The correct match between Item-I and Item -II is :

Item-I (drug)	Item-II (test)
A. Chloroxylenol	P. Carbylamine test
B. Norethindrone	Q. Sodium hydrogencarbonate test
C. Sulphapyridine	R. Ferric chloride test
D. Penicillin	S. Bayer's test

     (A) A $\to$ R; B $\to$ P; C $\to$ S; D $\to$ Q
     (B) A $\to$ Q; B $\to$ P; C $\to$ S; D $\to$ R
     (C) A $\to$ Q; B $\to$ S; C $\to$ P; D $\to$ R
     (D) A $\to$ R; B $\to$ S; C $\to$ P; D $\to$ Q

29. The increasing order of pKa of the following amino acids in aqueous solution is :
     Gly Asp Lys Arg
     (A) Arg < Lys < Gly < Asp
     (B) Asp < Gly < Lys < Arg
     (C) Asp < Gly < Arg < Lys
     (D) Gly < Asp < Arg < Lys

Space for rough use

30. Arrange the following amines in the decreasing order of basicity :
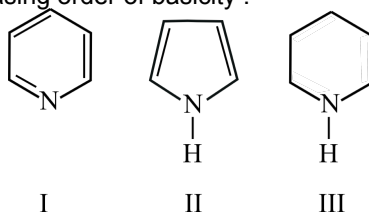
I   II   III

(A) I > II > III   (B) III > I > II   (C) III > II > I   (D) I > III > II

Mathematics

1. Let $A = \left\{ \theta \in \left(-\dfrac{\pi}{2}, \pi \right) : \dfrac{3 + 2i\sin\theta}{1 - 2i\sin\theta} \right.$ is purely imaginary $\}$. Then the sum of the elements in A is :

(A) $\dfrac{2\pi}{3}$ (B) $\dfrac{3\pi}{4}$ (C) π (D) $\dfrac{5\pi}{6}$

2. The equation of the line passing through $(-4, 3, 1),$ parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\dfrac{x+1}{-3} = \dfrac{y-3}{2} = \dfrac{z-2}{-1}$ is :

(A) $\dfrac{x+4}{3} = \dfrac{y-3}{-1} = \dfrac{z-1}{1}$ (B) $\dfrac{x+4}{-1} = \dfrac{y-3}{1} = \dfrac{z-1}{1}$

(C) $\dfrac{x-4}{2} = \dfrac{y+3}{1} = \dfrac{z+1}{4}$ (D) $\dfrac{x+4}{1} = \dfrac{y-3}{1} = \dfrac{z-1}{3}$

3. Axis of a parabola lies along x-axis. If its vertex and focus are at distance 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it ?

(A) $(5, 2\sqrt{6})$ (B) $(4, -4)$ (C) $(8, 6)$ (D) $(6, 4\sqrt{2})$

4. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is :

(A) 16   (B) 18   (C) 22   (D) 20

5. Let $a_1, a_2, \ldots, a_{30}$ be an A.P., $S = \displaystyle\sum_{i=1}^{30}$ and $T = \displaystyle\sum_{i=1}^{15} a(2i-1).$ If $a_5 = 27$ and $S - 2T = 75,$ then a_{10} is equal to :

(A) 42   (B) 52   (C) 57   (D) 47

6. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is :

(A) 300   (B) 350   (C) 200   (D) 500

7. Two cards are drawn successively with replacement from a well-shuffled deck of 53 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then $P(X = 1) + P(X = 2)$ equals :

(A) 25 / 169   (B) 24 / 169   (C) 49 / 169   (D) 52 / 169

8. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan\theta|$ is equal to:

(A) $\dfrac{8}{15}$ (B) $\dfrac{8}{17}$ (C) $\dfrac{4}{9}$ (D) $\dfrac{7}{17}$

Space for rough use

9. If $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \dfrac{\pi}{12}$, is equal to :

(A) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$
(B) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$
(C) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$
(D) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

10. If the Boolean expression
$(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \vee q$, where $\oplus,\ \odot \in \{\wedge, \vee\}$, then the ordered pair $(\oplus, \odot)$ is :

(A) $(\wedge, \vee)$
(B) $(\vee, \wedge)$
(C) $(\vee, \vee)$
(D) $(\wedge, \wedge)$

11. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to :

(A) -256
(B) 512
(C) -512
(D) 256

12. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is:

(A) $\sqrt{3}y = 3x + 1$
(B) $\sqrt{3}y = x + 3$
(C) $2\sqrt{3}y = 12x + 1$
(D) $2\sqrt{3}y = -x + 12$

13. For any $\theta \in \left(\dfrac{\pi}{4}, \dfrac{\pi}{2} \right)$, the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals:

(A) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$
(B) $13 - 4\cos^2\theta + 6\cos^4\theta$
(C) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$
(D) $13 - 4\cos^6\theta$

14. Three circles of radii $a, b, c\ (a < b < c)$ touch each other externally. If they have x-axis as a common tangent, then:

(A) $\dfrac{1}{\sqrt{a}} = \dfrac{1}{\sqrt{b}} + \dfrac{1}{\sqrt{c}}$
(B) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P.

(C) $\dfrac{1}{\sqrt{b}} = \dfrac{1}{\sqrt{a}} + \dfrac{1}{\sqrt{c}}$
(D) a, b, c are in A.P.

15. The system of linear equations
$x + y + z = 2$
$2x + 3y + 2z = 5$
$2x + 3y + (a^2 - 1)z = a + 1$

(A) has infinitely many solutions for $a = 4$
(B) has a unique solution for $|a| = \sqrt{3}$
(C) is inconsistent when $|a| = \sqrt{3}$
(D) is inconsistent when $a = 4$

16. If $y = y(x)$ is the solution of the differential equation, $x\dfrac{dy}{dx} + 2y = x^2$ satisfying $y(1) = 1$, then $y\left(\dfrac{1}{2}\right)$ is equal to:

(A) $\dfrac{13}{16}$
(B) $\dfrac{49}{16}$
(C) $\dfrac{1}{4}$
(D) $\dfrac{7}{64}$

17. The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y-axis also passes thorugh the point :

(A) $(-3, 0, -1)$
(B) $(3, 2, 1)$
(C) $(3, 3, -1)$
(D) $(-3, 1, 1)$

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18. Consider the set of all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true?

(A) The lines are all parallel.

(B) The lines are concurrent at the point $\left(\dfrac{3}{4}, \dfrac{1}{2}\right)$.

(C) Each line passes through the origin.

(D) The lines are not concurrent.

19. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a} \cdot \vec{c} = 4$, then $|\vec{c}|^2$ is equal to :

(A) 9
(B) $\dfrac{17}{2}$
(C) $\dfrac{19}{2}$
(D) 8

20. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point $(2, 3)$ to it and the y-axis is:

(A) $\dfrac{32}{3}$
(B) $\dfrac{56}{3}$
(C) $\dfrac{14}{3}$
(D) $\dfrac{8}{3}$

21. If the fractional part of the number $\dfrac{2^{403}}{15}$ is $\dfrac{k}{15}$, then k is equal to :

(A) 6
(B) 8
(C) 14
(D) 4

22. The value of $\displaystyle\int_0^\pi |\cos x|^3 dx$ is :

(A) $-\dfrac{4}{3}$
(B) $\dfrac{2}{3}$
(C) $\dfrac{4}{3}$
(D) 0

23. For $x^2 \neq n\pi + 1, n \in N$ (the set of natural numbers), the integral $\displaystyle\int x\sqrt{\dfrac{2\sin(x^2 - 1) - \sin 2(x^2 - 1)}{2\sin(x^2 - 1) + \sin 2(x^2 - 1)}}\,dx$ is equal to :

(A) $\log_e \left| \dfrac{1}{2} \sec^2(x^2 - 1) \right| + c$

(B) $\dfrac{1}{2}\log_e |\sec(x^2 - 1)| + c$

(C) $\log_e \left| \sec\left(\dfrac{x^2 - 1}{2}\right) \right| + c$

(D) $\dfrac{1}{2}\log_e \left| \sec^2\left(\dfrac{x^2 - 1}{2}\right) \right| + c$

24. If a, b and c be three distinct real numbers in G.P. and $a + b + c = xb$, then x cannot be:

(A) 2
(B) -3
(C) 4
(D) -2

25. $\displaystyle\lim_{y \to 0} \dfrac{\sqrt{1 + \sqrt{1 + y^4}} - \sqrt{2}}{y^4}$

(A) exists and equals $\dfrac{1}{2\sqrt{2}(\sqrt{2} + 1)}$

(B) exists and equals $\dfrac{1}{2\sqrt{2}}$

(C) does not exist

(D) exists and equals $\dfrac{1}{4\sqrt{2}}$

26. If $\cos^{-1}\left(\dfrac{2}{3x}\right) + \cos^{-1}\left(\dfrac{3}{4x}\right) = \dfrac{\pi}{2} \left(x > \dfrac{3}{4}\right)$, then x is equal to :

(A) $\dfrac{\sqrt{146}}{12}$
(B) $\dfrac{\sqrt{145}}{12}$
(C) $\dfrac{\sqrt{145}}{11}$
(D) $\dfrac{\sqrt{145}}{10}$

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27. Let $f : R \to R$ be a function defined as $\begin{cases} 5, & if & x \neq 1 \\ a + bx, & if & 1 < x < 3 \\ b + 5x, & if & 3 \leq x < 5 \\ 30 & if & x \geq 5 \end{cases}$

Then, f is :

(A) continuous if $a = -5$ and $b = 10$

(B) continuous if $a = 0$ and $b = 5$

(C) continuous if $a = 5$ and $b = 5$

(D) not continuous for any values of a and b

28. Let $0 < \theta < \dfrac{\pi}{2}$. If the eccentricity of the hyperbola $\dfrac{x^2}{\cos^2 \theta} - \dfrac{y^2}{\sin^2 \theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval :

(A) $(1, 3/2]$
(B) $(3/2, 2]$
(C) $(3, \infty)$
(D) $(2, 3]$

29. The maximum volume (in cu. m) of the right circular cone having slant height 3m is:

(A) $\dfrac{4}{3}\pi$
(B) 6π
(C) $3\sqrt{3}\pi$
(D) $2\sqrt{3}\pi$

30. For $x \in R - \{0, 1\}$, let $f_1(x) = \dfrac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \dfrac{1}{1-x}$ be three given functions. If a function, $J(x)$ satisfies $(f_2 . J . f_1)(x) = f_3(x)$ then $J(x)$ is equal to:

(A) $f_1(x)$
(B) $f_2(x)$
(C) $\dfrac{1}{x} f_3(x)$
(D) $f_3(x)$

Space for rough use