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PAPER-1 (B.E./B. TECH.) OF JEE (MAIN)

JEE (MAIN) 2018

CBT TEST PAPER

(WITH SOLUTION & ANSWER KEY)

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SUBJECT : PHYSICS, CHEMISTRY, MATHEMATICS

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PHYSICS

Straight Objective Type

This section contains **30 multiple choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. The relative uncertainty in the period of a satellite orbiting around the earth is 10^{-2}. If the relative uncertainty in the radius of the orbit is negligible the relative uncertainty in the mass of the earth is :

(1) 2×10^{-2} (2) 6×10^{-2} (3) 3×10^{-2} (4) 10^{-2}

Ans. **(1)**

Sol. From kepler's Law

$$T^2 = \frac{4\pi^2}{GM} r^3$$

$$\left|\frac{\Delta M}{M}\right| = 2\frac{\Delta T}{T} = 2 \times 10^{-2}$$

2. At some instant a radioactive sample S_1 having an activity 5 μCi has twice the number of nuclei as another sample S_2 which has an activity of 10 μCi. The half lives of S_1 and S_2 are :

(1) 5 years and 20 years, respectively (2) 20 years and 5 years, respectively
(3) 20 years and 10 years, respectively (4) 10 years and 20 years, respectively

Ans. **(1)**

Sol. Given : $N_1 = 2N_2$

$$\lambda_1 N_1 = \frac{\ell n2}{t_1} \times N_1 = 5\mu c_i$$

$$\lambda_2 N_2 = \frac{\ell n2}{t_2} \times N_2 = 10\mu c_i$$

$$\frac{t_2}{t_1} \times \frac{N_1}{N_2} = \frac{1}{2}$$

$$\frac{t_2}{t_1} = \frac{1}{4}$$

Hence 5years and 20 year

3. Two moles of helium are mixed with an moles of hydrogen. If $\frac{C_P}{C_V} = \frac{3}{2}$ for the mixture then the value of n is

(1) 1 (2) 3 (3) 2 (4) 3/2

Ans. **(3)**

Sol. $$\frac{C_p}{C_v} = \frac{f_{mix} + 2}{f_{mix}} = \frac{3}{2}$$

$\Rightarrow$ $f_{mix} = 4$

$$f_{mix} = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2}$$

$\Rightarrow$ $$4 = \frac{2 \times 3 + n_2 \times 5}{2 + n_2}$$ $\Rightarrow$ n_2 = 2mole

4. Unpolarized light of intensity I is incident on a system of two polarizers, A followed by B. The intensity of emergent light is I/2. If a third polarizer C is placed between A and B the intensity of emergent light is reduced to I/3. The angle between the polarizers A and C is θ, then

(1) $\cos\theta = \left(\frac{2}{3}\right)^{1/4}$ (2) $\cos\theta = \left(\frac{1}{3}\right)^{1/4}$ (3) $\cos\theta = \left(\frac{1}{3}\right)^{1/2}$ (4) $\cos\theta = \left(\frac{2}{3}\right)^{1/2}$

Ans. **(1)**

Sol. A and B have same alignment of transmission axis.

Lets assume c is introduced at θ angle

$$\frac{I}{2}\cos^2\theta \times \cos^2\theta = \frac{I}{3}$$

$\cos^4\theta = \frac{2}{3}$ $\Rightarrow$ $\cos\theta = \left(\frac{2}{3}\right)^{1/4}$

5. The de-Broglie wavelength (λ_B) associated with the electron orbiting in the second excited state of hydrogen atom is related to that in the ground state (λ_G) by :

(1) $\lambda_B = 3\lambda_G$ (2) $\lambda_B = 2\lambda_G$ (3) $\lambda_B = 3\lambda_{G/3}$ (4) $\lambda_B = 3\lambda_{G/2}$

Ans. **(1)**

Sol. $\frac{\lambda_B}{\lambda_G} = \frac{P_a}{P_B} = \frac{mv_G}{mv_B}$

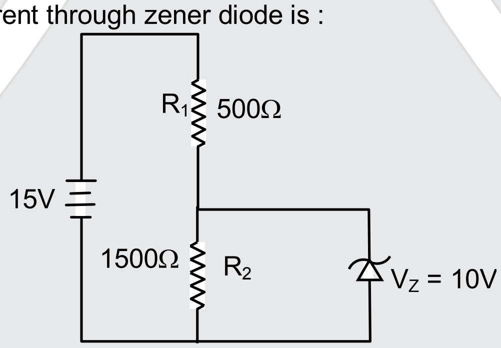
$V \times \frac{z}{n}$ So $\frac{\lambda_B}{\lambda_G} = \frac{n_B}{n_G} = \frac{3}{1}$

$\lambda_B = 3\lambda_G$

Length of Orbit = n × λ

$\lambda = \frac{2\pi r}{n}$ $\Rightarrow$ $\lambda \propto \frac{1}{n}$

6. In the given circuit the current through zener diode is :



(1) 3.3mA (2) 2.5mA (3) 5.5mA (4) 6.7mA

Ans. **(1)**

Sol. Current in $R_1 = I_1 = \frac{5}{500}$

$I_1 = 10$ mA

Current in $R_2 = I_2 = \frac{10}{1500}$ $\Rightarrow$ $I_2 = \frac{20}{3}$mA

Current in zener diode $= I_1 - I_2 = \left(10 - \frac{20}{3}\right)$mA $= \frac{10}{3}$mA

7. The end correction of a resonance column is 1cm. If the shortest length resonating with the tuning fork is 10cm, the next resonating length should be :

(1) 32cm (2) 40cm (3) 28cm (4) 36cm

Ans. **(1)**

Sol. Given : e = 1 cm

For first resonance

$$\frac{\lambda}{4} = \ell_1 + e = 11\text{cm}$$

For second resonance

$$\frac{3\lambda}{4} = \ell_1 + e \Rightarrow \quad \ell_2 = 3 \times 11 - 1 = 32 \text{ cm}$$

8. Two sitar strings A and B playing the note 'Dha' are slightly out of tune and produce beats of frequency 5Hz. The tension of the string B is slightly increased and the beat frequency is found to decrease by 3Hz. If the frequency of A is 425 Hz. the original frequency of B is :

(1) 428 Hz (2) 430 Hz (3) 422 Hz (4) 420 Hz

Ans. **(4)**

Sol. Frequency of B is either 420Hz or 430Hz As tension in B is increased its frequency will increase.

If frequency is 430Hz, beat frequency will increase

If frequency is 420 Hz beat frequency will decrease, hence correct answer is 420Hz

9. A power transmission line feeds input power at 2300V to a step down transformer with its primary windings having 4000 turns giving the output power at 230V. If the current in the primary of the transformer is 5A and its efficiency is 90% the output current would be :

(1) 45A (2) 50A (3) 20A (4) 25A

Ans. **(1)**

Sol. Efficiency n = 0.9 = $\frac{P_s}{P_P}$

$V_s I_s = 0.9 \times V_P I_P$

$I_s = \frac{0.9 \times 2300 \times 5}{230} = 45\text{A}$

10. A body of mass m starts moving from rest along x-axis so that its velocity varies as $\upsilon = a\sqrt{s}$ where a is a constant and s is the distance covered by the body. The total work done by all the forces acting on the body in the first t seconds after the start of the motion is :

(1) $8ma^4t^2$ (2) $\frac{1}{4}ma^4t^2$ (3) $4ma^4t^2$ (4) $\frac{1}{8}ma^4t^2$

Ans. **(4)**

Sol. $v = a\sqrt{s} = \frac{ds}{dt}$

$2\sqrt{s} = at$

$S = \frac{a^2t^2}{4}$

$F = m \times \frac{a^2}{2}$

Work = $\frac{ma^2}{2} \times \frac{a^2t^2}{4} = \frac{1}{8}ma^4t^2$

11. Suppose that the angular velocity of rotation of earth is increased. Then as a consequence:
(1) Weight of the object every where on the earth will decrease
(2) Weight of the object every where on the earth will increase
(3) Except at poles weight of the object on the earth will decrease
(4) There will be no change in weight anywhere on the earth.

Ans. **(3)**

Sol. $g' = g - \omega^2 R\cos^2\phi$
Where ϕ is latitude there will be no change in gravity at poles as $\phi = 90°$
At all other points as ω increases g' will decrease.

12. Both the nucleus and the atom of some element are in their respective first excited states. They get de-excited by emitting photons of wavelengths λ_N, λ_A respectively. The ratio $\frac{\lambda_N}{\lambda_A}$ is closest to :
(1) 10^{-1} (2) 10^{-6} (3) 10 (4) 10^{-10}

Ans. **(2)**

Sol. $\frac{\lambda_N}{\lambda_a} = \frac{E_a}{E_N}$
where E_a and E_N are energies of photons from atom and nucleus respectively. E_N is of the order of MeV and E_a in few eV.
So $\frac{\lambda_N}{\lambda_a} = 10^{-6}$

13. A plane electromagnetic wave of wavelength λ has an intensity I. It is propagating along the positive Y-direction. The allowed expressions for the electric and magnetic fields are given by :

(1) $\vec{E} = \sqrt{\frac{2I}{\varepsilon_0 c}}\cos\left[\frac{2\pi}{\lambda}(y - ct)\right]\hat{k}$; $\vec{B} = +\frac{1}{c}E\hat{i}$

(2) $\vec{E} = \sqrt{\frac{I}{\varepsilon_0 c}}\cos\left[\frac{2\pi}{\lambda}(y - ct)\right]\hat{k}$; $\vec{B} = +\frac{1}{c}E\hat{i}$

(3) $\vec{E} = \sqrt{\frac{2I}{\varepsilon_0 c}}\cos\left[\frac{2\pi}{\lambda}(y - ct)\right]\hat{k}$; $\vec{B} = \frac{1}{c}E\hat{i}$

(4) $\vec{E} = \sqrt{\frac{I}{\varepsilon_0 c}}\cos\left[\frac{2\pi}{\lambda}(y - ct)\right]\hat{i}$; $\vec{B} = \frac{1}{c}E\hat{k}$

Ans. **(1)**

Sol. If E_0 is magnitude of electric field then $\frac{1}{2}\varepsilon_0 E^2 \times C = I$

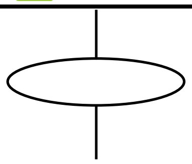
$$E_0 = \sqrt{\frac{2I}{C\varepsilon_0}}$$

$$B_0 = \frac{E_0}{C}$$

direction of $\vec{E}\times\vec{B}$ will be along $+\hat{j}$.

14. A charge q is spread uniformly over an insulated loop of radius r. If it is rotated with an angular velocity ω with respect to normal axis then magnetic moment of the loop is :
(1) $\frac{3}{2}q\omega r^2$ (2) $\frac{1}{2}q\omega r^2$ (3) $q\omega r^2$ (4) $\frac{4}{3}q\omega r^2$

Ans. **(2)**

Sol. 

$\frac{M}{L} = \frac{q}{2m}$

$M = \frac{q}{2m} \times mr^2\omega$

$M = \frac{q\omega r^2}{2}$

15. A heating element has a resistance of 100Ω at room temperature. When it is connected to a supply of 220V a steady current of 2A passes in it and temperature is 500°C more than room temperature. What is the temperature coefficient of resistance of the heating element?

(1) 5×10^{-4} °C^{-1} (2) 2×10^{-4} °C^{-1} (3) 1×10^{-4} °C^{-1} (4) 0.5×10^{-4} °C^{-1}

Ans. **(2)**

Sol. Resistance after temperature increases by 500°C = $\frac{220}{2}$ = 110Ω

$110 = 100 (1 + \alpha 500)$

$\alpha = \frac{10}{100 \times 500}$

$\alpha = 2 \times 10^{-4}$ °C^{-1}

16. A coil of cross-sectional area A having n turns is placed in a uniform magnetic field B. When it is rotated with an angular velocity ω the maximum e.m.f. induced in the coil will be :

(1) $\frac{3}{2}$ nBAω (2) 3nBAω (3) nBAω (4) $\frac{1}{2}$ nBAω

Ans. **(3)**

Sol. $\varepsilon = BA\omega n \sin\omega t$

$\varepsilon_{max} = BA\omega n$

17. A ray of light is incident at an angle of 60° on one face of a prism of angle 30°. The emergent ray of light makes an angle of 30° with incident ray. The angle made by the emergent ray with second face of prism will be :

(1) 0° (2) 90° (3) 30° (4) 45°

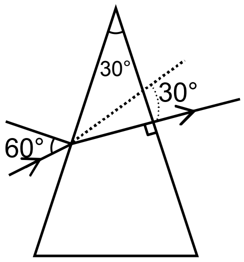
Ans. **(2)**

Sol. $\delta = I + e - A$

$30 = 60 + e - 30°$

$\Rightarrow$ e = 0

So angle with face = 90°



18. A galvanometer with its coil resistance 25Ω requires a current of 1mA for its full deflection. In order to construct an ammeter to read up to a current of 2A the approximate value of the shunt resistance should be :

(1) $1.25 \times 10^{-2}\Omega$ (2) $2.5 \times 10^{-3}\Omega$ (3) $2.5 \times 10^{-2}\Omega$ (4) $1.25 \times 10^{-3}\Omega$

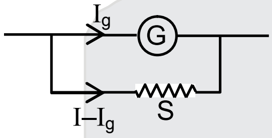
Ans. **(1)**

Sol. $I_g R_g = (I - I_g)S$

$$S \simeq \frac{10^{-3} \times 25}{2}$$

$S \simeq 12.5 \times 10^{-3}$

or $1.25 \times 10^{-2}\Omega$



19. An oscillator of mass M is at rest in the equilibrium position in a potential $V = \frac{1}{2}k(x - X)^2$. A particle of mass m comes from right with speed u and collides completely inelastically with M and sticks to it. This process repeats every time the oscillator crosses its equilibrium position. The amplitude of oscillations after 13 collisions is : (M = 10, m = 5, u = 1, k = 1)

(1) $\frac{2}{3}$ (2) $\frac{1}{\sqrt{3}}$ (3) $\sqrt{\frac{3}{5}}$ (4) $\frac{1}{2}$

Ans. **(2)**

Sol. In first collision mu momentum will be imparted to system. In second collision when momentum of (M + m) is in opposite direction mu momentum of particle will make its momentum zero.

on 13th collision

$\boxed{m} \longrightarrow \qquad \boxed{M + 12m}$

$\boxed{M + 13m} \longrightarrow V$

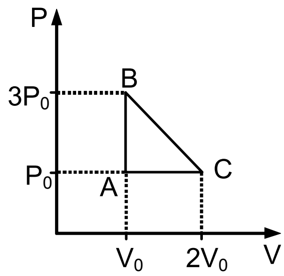
$mu = (M + 13m)v$

$$v = \frac{mu}{M + 13m} = \frac{u}{15}$$

$v = \omega A$

$$\Rightarrow \quad \frac{u}{15} = \sqrt{\frac{K}{M + 13m}} \times A \quad \Rightarrow \quad A = \frac{1}{15}\sqrt{\frac{75}{1}} = \frac{1}{\sqrt{3}}$$

20. One mole of an ideal monatomic gas is taken along the path ABCA as shown in the PV diagram. The maximum temperature attained by the gas along the path BC is given by :



(1) $\frac{25}{4}\frac{P_0V_0}{R}$ (2) $\frac{5}{8}\frac{P_0V_0}{R}$ (3) $\frac{25}{8}\frac{P_0V_0}{R}$ (4) $\frac{25}{16}\frac{P_0V_0}{R}$

Ans. **(3)**

Sol. Equation of line BC

$$P = P_0 - \frac{2P_0}{V_0}(V - 2V_0)$$

$$\text{Temperature} = \frac{P_0V - \frac{2P_0V^2}{V_0} + 4P_0V}{1 \times R}$$

$$T = \frac{P_0}{R}\left[5V - \frac{2V^2}{V_0}\right]$$

$$\frac{dT}{dV} = 0 \quad \Rightarrow \quad 5 - \frac{4V}{V_0} = 0 \quad \Rightarrow \quad V = \frac{5}{4}V_0$$

$$T = \frac{P_0}{R}\left[5 \times \frac{5V_0}{4} - \frac{2}{V_0} \times \frac{25}{16}V_0^2\right]$$

$$T = \frac{25}{8}\frac{P_0V_0}{R}$$

21. In a circuit for finding the resistance of a galvanometer by half deflection method a 6V battery and a high resistance of 11kΩ are used. The figure of merit of the galvanometer produces a deflection of $\theta = 9$ divisions when current flows in the circuit. The value of the shunt resistance that can cause the deflection of $\theta/2$ is

(1) 550Ω (2) 220Ω (3) 55Ω (4) 110Ω

Ans. **(4)**

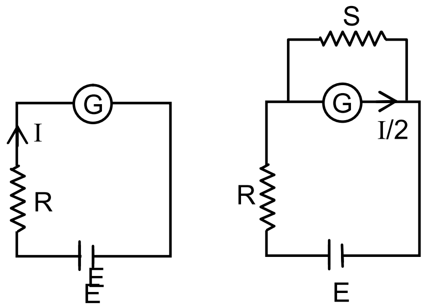
Sol. $I = \frac{\varepsilon}{R + G}$ $G = \frac{1}{9}K\Omega$

$$\frac{I}{2} = \frac{\varepsilon}{R + \frac{GS}{G+S}} \times \frac{S}{S+G} \quad \Rightarrow \quad \frac{I}{2} = \frac{\varepsilon S}{R(S+G) + GS}$$

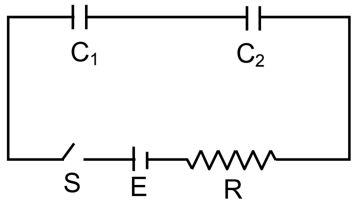
$$S = \frac{RG \times \frac{I}{2}}{\varepsilon - \frac{(R+G)I}{2}}$$

$$S = \frac{11 \times 10^3 \times \frac{1}{9} \times 10^3 \times 270 \times 10^{-6}}{6 - \left(\frac{6}{2}\right)}$$

S = 110Ω



22. In the following circuit the switch S is closed at t = 0. The charge on the capacitor C_1 as a function of time will be given by $\left(C_{eq} = \dfrac{C_1C_2}{C_1+C_2}\right)$

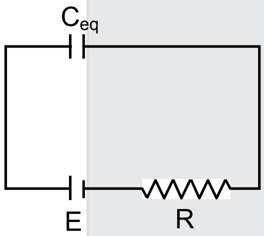


(1) $C_1E[1 - \exp(-tR/C_1)]$
(2) $C_{eq}E \exp(-t/RC_{eq})$
(3) $C_{eq}E [1 - \exp(-t/RC_{eq})]$
(4) $C_2E[1 - \exp(-t/RC_2)]$

Ans. **(3)**

Sol. $q = C_{eq}\, E \left[1 - e^{-t/Rc_{eq}}\right]$

Both capacitor will have same charge as they are connected in series.



23. Let $\vec{A} = (\hat{i} + \hat{j})$ and $\vec{B} = (2\hat{i} - \hat{j})$. The magnitude of a coplanar vector $\vec{C}$ such that $\vec{A}.\vec{C} = \vec{B}.\vec{C} = \vec{A}.\vec{B}$ is given by :

(1) $\sqrt{\dfrac{9}{12}}$ (2) $\sqrt{\dfrac{20}{9}}$ (3) $\sqrt{\dfrac{5}{9}}$ (4) $\sqrt{\dfrac{10}{9}}$

Ans. **(3)**

Sol. If $\vec{C} = a\hat{i} + b\hat{j}$ then

$\vec{A}.\vec{C} = \vec{A}.\vec{B}$

$a + b = 1$ ...(i)

$\vec{B}.\vec{C} = \vec{A}.\vec{B}$

$2a - b = 1$ ...(ii)

Solving equation (i) and (ii) we get

$a = \dfrac{1}{3}, b = \dfrac{2}{3}$

$|\vec{C}| = \sqrt{\dfrac{1}{9} + \dfrac{4}{9}} = \sqrt{\dfrac{5}{9}}$

24. A particle executes simple harmonic motion and is located at x = a, b and c at times t_0, $2t_0$ and $3t_0$ respectively. The frequency of the oscillation is :

(1) $\dfrac{1}{2\pi t_0}\cos^{-1}\left(\dfrac{a+c}{2b}\right)$ (2) $\dfrac{1}{2\pi t_0}\cos^{-1}\left(\dfrac{a+2b}{3c}\right)$ (3) $\dfrac{1}{2\pi t_0}\cos^{-1}\left(\dfrac{a+b}{2c}\right)$ (4) $\dfrac{1}{2\pi t_0}\cos^{-1}\left(\dfrac{2a+3c}{b}\right)$

Ans. **(1)**

Sol. $a = A\sin\omega t_0$

$b = A\sin2\omega t_0$

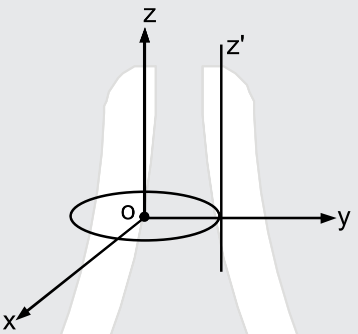
$c = A\sin3\omega t_0$

$a + c = A[\sin\omega t_0 + \sin3\omega t_0] = 2A\sin2\omega t_0\cos\omega t_0$

$\frac{a+c}{b} = 2\cos\omega t_0$

$\omega = \frac{1}{t_0}\cos^{-1}\left(\frac{a+c}{2b}\right) \quad \Rightarrow \quad f = \frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$

25. A thin circular disk is in the xy plane as shown in the figure. The ratio of its moment of inertia about z and z' axes will be :



(1) 1 : 4 (2) 1 : 5 (3) 1 : 3 (4) 1 : 2

Ans. **(3)**

Sol. $I_z = \frac{mR^2}{2}$

$I_z' = \frac{3}{2}mR^2 \quad \frac{I_z}{I_z'} = \frac{1}{3}$

26. Two identical conducting spheres A and B carry equal charge. They are separated by a distance much larger than their diameters and the force between them is F. A third identical conducting sphere C is uncharged. Sphere C is first touched to A then to B and then removed. As a result the force between A and B would be equal to :

(1) $\frac{3F}{4}$ (2) $\frac{F}{2}$ (3) $\frac{3F}{8}$ (4) F

Ans. **(3)**

Sol.

$F = \frac{kq^2}{r^2}$ when A and C are touched charge on both will be $\frac{q}{2}$

Then when B and C are touched

$q_B = \frac{\frac{q}{2}+q}{2} = \frac{3q}{4}$

$F' = \frac{kq_Aq_B}{r^2} = \frac{k\times\frac{q}{2}\times\frac{3q}{4}}{r^2} = \frac{3}{8}\frac{kq^2}{r^2} = \frac{3}{8}F$

27. Two particles of the same mass m are moving in circular orbits because of force given by $F(r) = \frac{-16}{r} - r^3$. The first particle is at distance r = 1 and the second at r = 4. The best estimate for the ratio of kinetic energies of the first and the second particle is closest to :

(1) 3×10^{-3} (2) 6×10^{2} (3) 6×10^{-2} (4) 10^{-1}

Ans. **(3)**

Sol. $\frac{mV^2}{r} = \frac{16}{r} + r^3$

$KE_0 = \frac{1}{2} mV^2$

$= \frac{1}{2}[16 + r^4]$

$\frac{K_1}{K_2} = \frac{\frac{16+1}{2}}{\frac{16+256}{2}} = \frac{17}{272}$

$\frac{K_1}{K_2} \simeq 6 \times 10^{-2}$

28. The percentage errors in quantities P, Q, R and S are 0.5%, 1%, 3% and 1.5% respectively in the measurement of a physical quantity $A = \frac{P^3Q^2}{\sqrt{RS}}$. The maximum percentage error in the value of A will be :

(1) 6.5% (2) 7.5% (3) 6.0% (4) 8.5%

Ans. **(1)**

Sol. $\frac{\Delta A}{A} = \frac{3\Delta P}{P} + \frac{2\Delta Q}{Q} + \frac{1}{2}\frac{\Delta R}{R} + \frac{\Delta S}{S}$

$= 3 \times 0.5 + 2 \times 1 + \frac{1}{2} \times 3 + 1.5$

$= 1.5 + 2 + 1.5 + 1.5$

$\frac{\Delta A}{A} = 6.5\%$

29. A carrier wave of peak voltage 14V is used for transmitting a message signal given to achieve a modulation index of 80% will be :

(1) 22.4V (2) 7V (3) 11.2V (4) 28V

Ans. **(3)**

Sol. $m = \frac{A_m}{A_c}$

$A_m = 0.8 \times 14$

$= 11.2V$

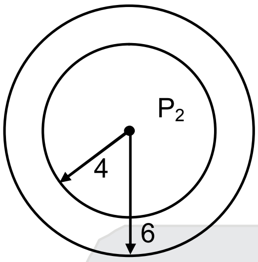
30. A small soap bubble of radius 4cm is trapped inside another bubble of radius 6cm without any contact. Let P_2 be the pressure inside the inner bubble and P_0 the pressure outside the outer bubble. Radius of another bubble with pressure difference $P_2 - P_0$ between its inside and outside would be :

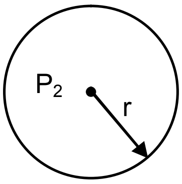
(1) 2.4cm (2) 12cm (3) 4.8cm (4) 6cm

Ans. **(1)**

Sol. $P_2 = P_0 + \frac{4T}{6} + \frac{4T}{4}$ $\qquad P_2 = P_0 + \frac{4T}{r}$

$\Rightarrow \quad \frac{1}{r} = \frac{1}{6} + \frac{1}{4} \qquad \Rightarrow \quad r = 2.4\text{cm}$





CHEMISTRY

Straight Objective Type

This section contains **30 multiple choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. For standardizing NaOH solution, which of the following is used as a primary standard ?

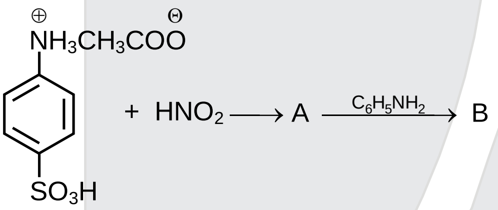
(1) Sodium tetraborate (2) Ferrous Ammonium Sulfate

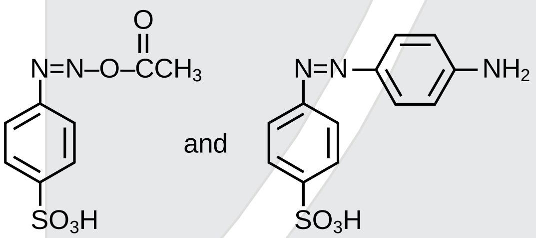
(3) Oxalic acid (4) dil. HCl

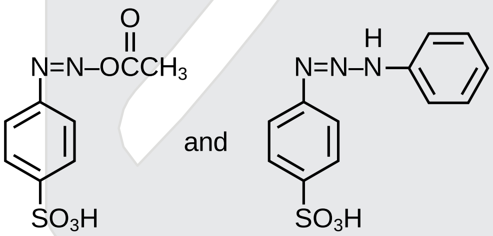
Ans. (3)

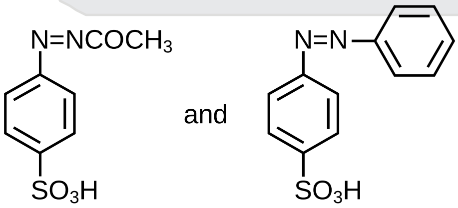
Sol. Oxalic acid is used as a primary standard for NaOH standardizing.

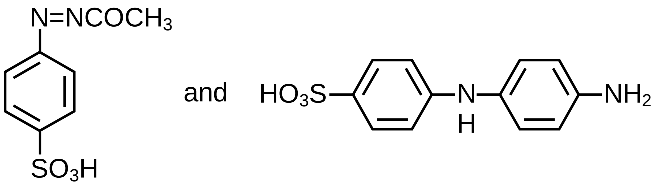
2. Products A and B formed in the following reactions are respectively :



(1) 

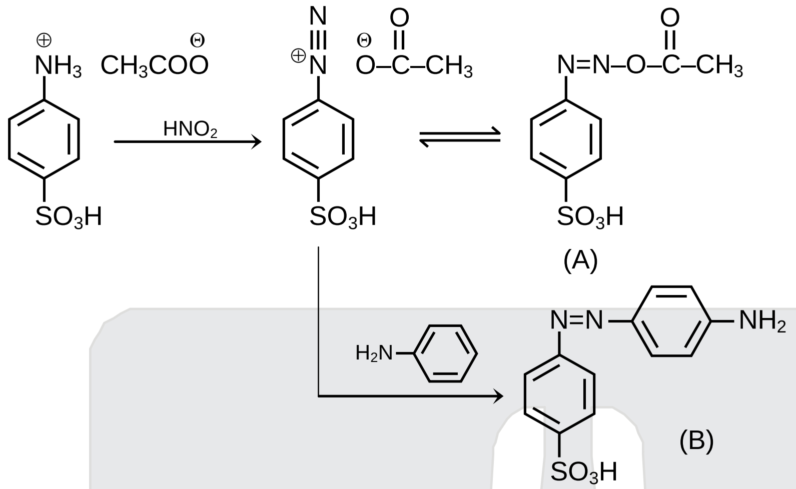
(2) 

(3) 

(4) 

Ans. (1)

Sol.



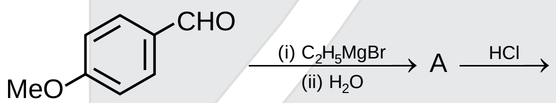
3. When XO_2 is fused with an alkali metal hydroxide in presence of an oxidizing agent such as KNO_3 ; a dark green product is formed which disproportioates in acidic solution to afford a dark purple solution. X is :

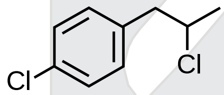
(1) Mn (2) Cr (3) V (4) Ti

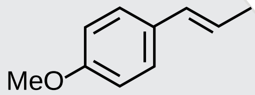
Ans. (1)

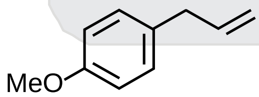
Sol. $MnO_2 + KOH \longrightarrow \underset{\text{(dark green)}}{K_2MnO_4} \xrightarrow[\text{solution}]{\text{Acidic}} \underset{\text{(dark purple)}}{KMnO_4}$

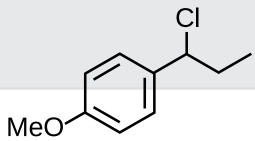
4. The major product B formed in the following reaction sequence is :



(1) 

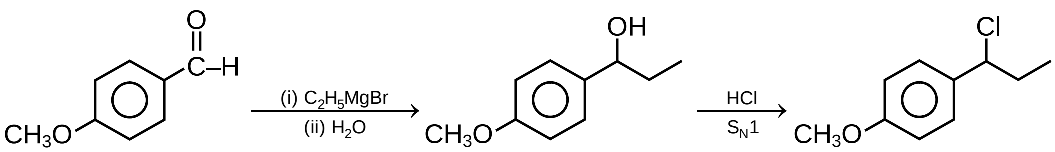
(2) 

(3) 

(4) 

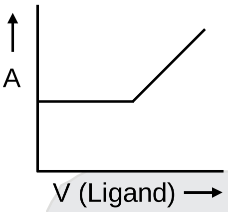
Ans. (4)

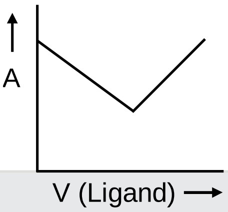
Sol.

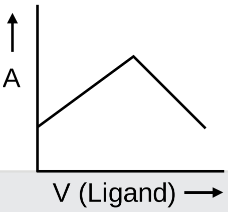


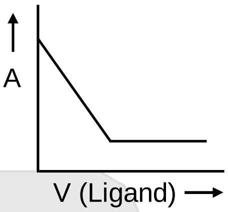
5. In a complexometric titration of metal ion with ligand

M (Metal ion) + L (Ligand) $\rightarrow$ C (Complex) end point is estimated spectrophotometrically (through light absorption). If 'M' and 'C' do not absorb light and only 'L' absorbs, then the titration plot between absorbed light (A) versus volume of ligand 'L' (V) would look like :

(1) 

(2) 

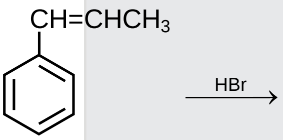
(3) 

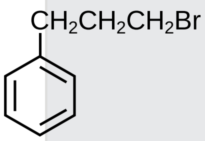
(4) 

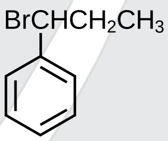
Ans. (1)

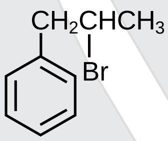
Sol. Initially ligand consumed by metal due to formation of complex. So absorbed light (A) remain constant, after complex formation is completed, extra volume of ligand solution increases ligand concentration and also increases absorbed light.

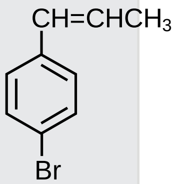
6. The major product of the following reaction is :



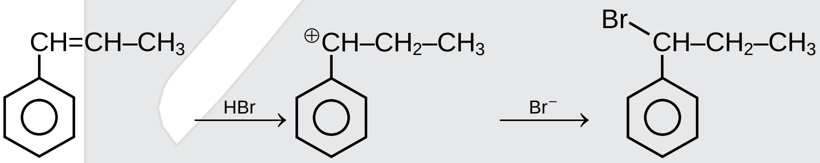
(1) 

(2) 

(3) 

(4) 

Ans. (2)

Sol. 

7. Among the following, the **incorrect** statement is :

(1) Cellulose and amylose has 1,4-glycosidic linkage.

(2) Lactose contains β-D-galactose and β-D-glucose.

(3) Maltose and lactose has 1,4-glycosidic linkage.

(4) Sucrose and amylose has 1,2-glycosidic linkage.

Ans. (4)

Sol. In amylose 1,4-glycosidic linkage is present.

8. In the extraction of copper from its sulphide ore, metal is finally obtained by the oxidation of cuprous sulphide with :

(1) SO_2 (2) Fe_2O_3 (3) Cu_2O (4) CO

Ans. (3)

Sol. $Cu_2S + 2Cu_2O \longrightarrow 6Cu + SO_2$

9. Among the oxides of nitrogen :

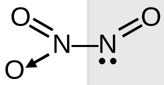
N_2O_3, N_2O_4 and N_2O_5 ; the molecule(s) having nitrogen-nitrogen bond is/are :

(1) N_2O_3 and N_2O_4 (2) N_2O_4 and N_2O_5 (3) N_2O_3 and N_2O_5 (4) Only N_2O_5

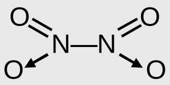
Ans. (1)

Sol.

N_2O_3



N_2O_4

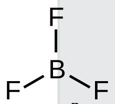


10. Which of the following conversions involves change in both shape and hybridisation ?

(1) $H_2O \rightarrow H_3O^+$ (2) $BF_3 \rightarrow BF_4^-$ (3) $CH_4 \rightarrow C_2H_6$ (4) $NH_3 \rightarrow NH_4^+$

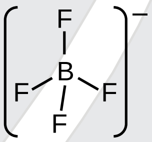
Ans. (2)

Sol. $BF_3 \longrightarrow BF_4^-$



sp^2

Triangle planar

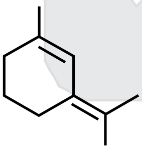


sp^3

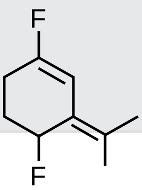
Tetrahedral

11. The most polar compound among the following is :

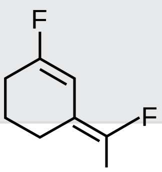
(1)



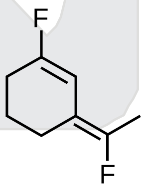
(2)



(3)

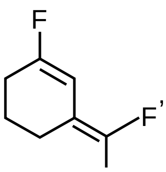


(4)



Ans. (3)

Sol. In



, the bond dipole vector of C–F bond is not subtractive.

12. In Wilkinson's catalyst, the hybridization of central metal ion and its shape are respectively :

(1) sp^3d, trigonal bipyramidal (2) d^2sp^3, octahedral

(3) dsp^2, square planar (4) sp^3, tetrahedral

Ans. (3)

Sol. Wilkinson catalyst

$[RhCl(PPh_3)_3]$

13. At 320 K, a gas A_2 is 20 % dissociated to A(g). The standard free energy change at 320 K and 1 atm in J mol^{-1} is approximately : (R = 8.314 JK^{-1} mol^{-1} ; ln 2 = 0.693 ; ln 3 = 1.098)

(1) 1844 (2) 2068 (3) 4281 (4) 4763

Ans. (3)

Sol. $A_2(g) \rightleftharpoons 2A(g)$

$1 \qquad 0$

$1 - 1\times\frac{20}{100} \qquad 2\times\frac{20}{100}$

$0.8 \qquad 0.4$

$$K_p = \frac{(p_A)^2}{(p_{A_2})} = \frac{0.4\times0.4}{0.8} = 0.2$$

$\Delta G^o = -2.303 \times 8.314 \times 320 \log_{10} 0.2 = 4281$ J/mole

14. Which of the following complexes will show geometrical isomerism ?

(1) Potassium tris(oxalato)chromate(III)

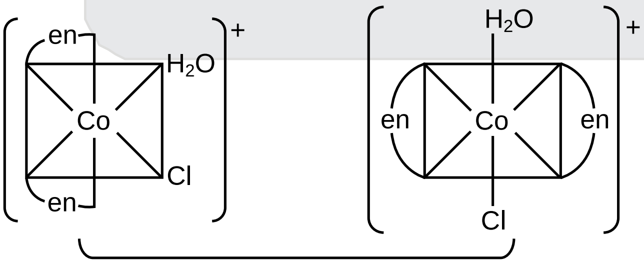
(2) Pentaaquachlorochromium(III)chloride

(3) Aquachlorobis(ethylenediamine)cobalt(II) chloride

(4) Potassium amminetrichloroplatinate(II)

Ans. (3)

Sol. $[Co(H_2O)Cl(en)_2]Cl$



(Geometrical Isomer)

15. Which of the following statements is **false** ?

(1) Splitting of spectral lines in electrical field is called Stark effect.

(2) Frequency of emitted radiation from a black body goes from a lower wavelength of higher wavelength as the temperature increases.

(3) Photon has momentum as well as wavelength.

(4) Rydberg constant has unit of energy.

Ans. (2) and (4) [both are false]

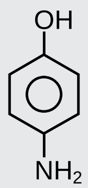
Sol. When temperature is increased, black body emit high energy radiation, from higher wavelength to lower wavelength.

Rydberg constant has unit length^{-1} (i.e. cm^{-1})

16. When 9.65 ampere current was passed for 1.0 hour into nitrobenzene in acidic medium, the amount of p-aminophenol produced is :

(1) 109.0 g (2) 98.1 g (3) 9.81 g (4) 10.9 g

Ans. (3)

Sol. $C_6H_5NO_2 \longrightarrow$ 

$$4e^- + 4H^+ + C_6H_5NO_2 \longrightarrow \underset{\text{M.W. = 109 g}}{C_6H_4(OH)(NH_2)} + H_2O$$

(v.f.) = 4 $\quad W = ZIt = \dfrac{E}{F} \times I \times t \quad \left(E = \dfrac{M}{4}\right)$

$$W = \frac{109 \times 9.65 \times 60 \times 60}{4 \times 96500}$$

W = 9.81 g

17. For which of the following processes, ΔS is negative ?

(1) C(diamond) $\rightarrow$ C(graphite) (2) N_2(g, 1 atm) $\rightarrow$ N_2(g, 5 atm)

(3) N_2(g, 273 K) $\rightarrow$ N_2(g, 300 K) (4) H_2(g) $\rightarrow$ 2H(g)

Ans. (2)

Sol. N_2 (g, 1 atm) $\longrightarrow$ N_2 (g, 5 atm)

$$\Delta S = \left(nC_p \ln\frac{T_2}{T_1}\right) + nR \ln\frac{V_2}{V_1} \quad \text{for isothermal process } T_1 = T_2 \text{ and } \frac{V_2}{V_1} = \frac{P_1}{P_2}$$

$$= 0 + nR \ln\frac{P_1}{P_2} = nR \ln\frac{1}{5}$$

$\Delta S < 0$

18. An unknown chlorohydrocarbon has 3.55 % of chlorine. If each molecule of the hydrocarbon has one chlorine atom only ; chlorine atoms present in 1 g of chlorohydrocarbon are :

(Atomic wt. of Cl = 35.5 u ; Avogadro constant = 6.023×10^{23} mol^{-1})

(1) 6.023×10^{9} (2) 6.023×10^{23} (3) 6.023×10^{21} (4) 6.023×10^{20}

Ans. (4)

Sol. C_xH_yCl

% Cl = 3.55

Weight of Cl = $1 \times \frac{3.55}{100}$

$n_{Cl^-} = \frac{1 \times 3.55}{100 \times 35.5}$

No of Cl^- ion = $\frac{1 \times 3.55}{100 \times 35.5} \times 6.023 \times 10^{23}$

$= 6.023 \times 10^{20}$

19. The **incorrect** statement is :

(1) Cu^{2+} ion gives chocolate coloured precipitate with potassium ferrocyanide solution.

(2) Cu^{2+} and Ni^{2+} ions give black precipitate with H_2S in presence of HCl solution.

(3) Ferric ion gives blood red colour with potassium thiocyanate.

(4) Cu^{2+} salts give red coloured borax bead test in reducing flame.

Ans. (2)

Sol. Due to common ion effect, sufficient S^{2-} concentration not produce and not formed ppt of NiS.

20. The mass of a non-volatile, non-electrolyte solute (molar mass = 50 g mol^{-1}) needed to be dissolved in 114 g octane to reduce its vapour pressure to 75 %, is :

(1) 37.5 g (2) 75 g (3) 150 g (4) 50 g

Ans. (Bonus)

Sol. $\frac{P^o - P_s}{P_s} = \frac{n}{N}$

$$\frac{100P - 75P}{75P} = \frac{\frac{W}{50}}{1}$$

$$\frac{25}{75} = \frac{W}{50}$$

$$W = \frac{50}{3} \text{ g}$$

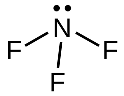
21. The **incorrect** geometry is represented by :

(1) NF_3 – trigonal planar (2) BF_3 – trigonal planar

(3) AsF_5 – trigonal bipyramidal (4) H_2O – bent

Ans. (1)

Sol. NF_3



sp^3

Trigonal pyramidal

22. Assuming ideal gas behaviour, the ratio of density of ammonia to that of hydrogen chlroide at same temperature and pressure is : (Atomic wt. of Cl 35.5 u)

(1) 1.46 (2) 1.64 (3) 0.46 (4) 0.64

Ans. (3)

Sol. $d = \dfrac{P(M.w.)}{RT}$

$$\frac{d_{NH_3}}{d_{HCl}} = \frac{(M.w.)_{NH_3}}{(M.w.)_{HCl}} = \frac{17}{36.5} = 0.46$$

23. The correct match between items of List-I and List-II is :

	List-I		List-II
(A)	Phenelzine	(P)	Pyrimidine
(B)	Chloroxylenol	(Q)	Furan
(C)	Uracil	(R)	Hydrazine
(D)	Ranitidine	(S)	Phenol

(1) (A)-(S), (B)-(R), (C)-(Q), (D)-(P) (2) (A)-(R), (B)-(S), (C)-(P), (D)-(Q)

(3) (A)-(R), (B)-(S), (C)-(Q), (D)-(P) (4) (A)-(S), (B)-(R), (C)-(P), (D)-(Q)

Ans. (2)

Sol. $\rightarrow$ Phenelzine contains hydrazine

$\rightarrow$ Chloroxylenol contains phenol

$\rightarrow$ Uracil is the pyrimidine base

$\rightarrow$ Ranitidine contains furan ring

24. The gas phase reaction $2NO_2(g) \rightarrow N_2O_4(g)$ is an exothermic reaction. The decomposition of N_2O_4, in equilibrium mixture of $NO_2(g)$ and $N_2O_4(g)$, can be increased by :

(1) addition of an inert gas at constant pressure.

(2) lowering the temperature

(3) increasing the pressure

(4) addition of an inert gas at constant volume.

Ans. (1)

Sol. $2NO_2(g) \longrightarrow N_2O_4(g)$ $\Delta H = (–)$

By addition of an inert gas at constant pressure, volume increases, so reaction moving in backward direction and decomposition of N_2O_4 increases.

25. Which one of the following is not a property of physical adsorption ?

(1) Higher the pressure, more the adsorption

(2) Greater the surface area, more the adsorption

(3) Lower the temperature, more the adsorption

(4) Unilayer adsorption occurs

Ans. (4)

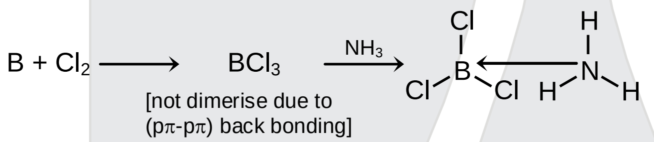
Sol. Physical adsorption is multilayer adsorption.

26. A group 13 element 'X' reacts with chlorine gas to produce a compound XCl_3. XCl_3 is electron deficient and easily reacts with NH_3 to form $Cl_3X \leftarrow NH_3$ adduct; however, XCl_3 does not dimerize. X is :

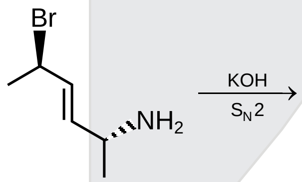
(1) B (2) Al (3) In (4) Ga

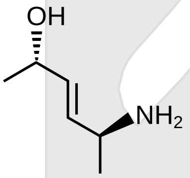
Ans. (1)

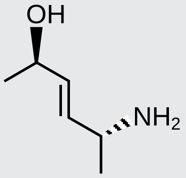
Sol. BCl_3

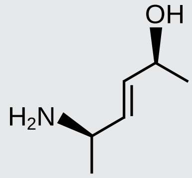


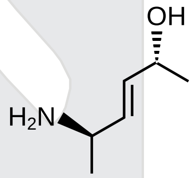
27. The major product of the following reaction is :



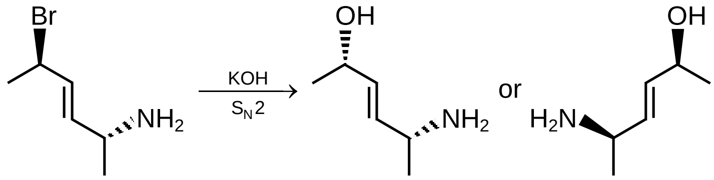
(1) 

(2) 

(3) 

(4) 

Ans. (3)

Sol. 

Inversion takes place at the carbon containing bromine atom.

28. If 50 % of a reaction occurs in 100 second and 75 % of the reaction occurs in 200 second, the order of this reaction is :

(1) 2 (2) 3 (3) Zero (4) 1

Ans. (4)

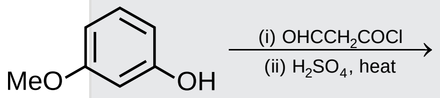
Sol.

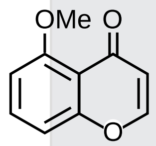
$$C_o \xrightarrow{100\text{ second}} \frac{C_o}{2} \xrightarrow{200\text{ second}} \frac{C_o}{4}$$

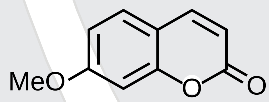
75 % complete

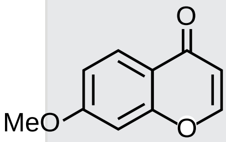
First order reaction as half life is constant.

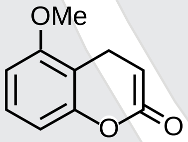
29. The major product of the following reaction is :



(1) 

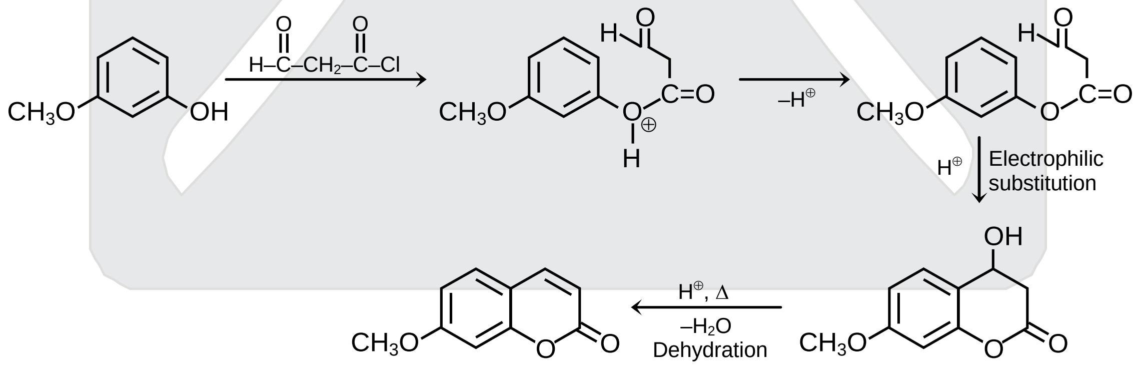
(2) 

(3) 

(4) 

Ans. (2)

Sol.

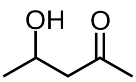


30. Which of the following compounds will most readily be dehydrated to give alkene under acidic condition?

(1) 4-Hydroxypentan-2-one (2) 3-Hydroxypentan-2-one

(3) 1-Pentanol (4) 2-Hydroxycyclopentanone

Ans. (1)

Sol. 

will most readily be dehydrated to give conjugated alkene.

MATHEMATICS
PART– C

Straight Objective Type (सीधे वस्तुनिष्ठ प्रकार)

This section contains **30 multiple choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

इस खण्ड में **30 बहु–विकल्पी प्रश्न** हैं। प्रत्येक प्रश्न के 4 विकल्प (1), (2), (3) तथा (4) हैं, जिनमें से **सिर्फ एक सही** है।

1. If $x = \sqrt{2^{\operatorname{cosec}^{-1}t}}$ and $y = \sqrt{2^{\sec^{-1}t}}$ $(|t| \geq 1)$, then $\frac{dy}{dx}$ is equal to :

(1) $\frac{y}{x}$ (2*) $-\frac{y}{x}$ (3) $-\frac{x}{y}$ (4) $\frac{x}{y}$

Ans. **(2)**

Sol. $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{1}{2\sqrt{2^{\sec^{-1}t}}}\,2^{\sec^{-1}t}\,\ell n2\left(\frac{1}{t\sqrt{t^2-1}}\right)}{\frac{1}{2\sqrt{2^{\operatorname{cosec}^{-1}t}}}\,2^{\operatorname{cosec}^{-1}t}\,\ell n2\left(\frac{1}{t\sqrt{t^2-1}}\right)}$$

$$= -\frac{\sqrt{2^{\sec^{-1}t}}}{\sqrt{2^{\operatorname{cosec}^{-1}t}}} = -\frac{y}{x}$$

2. Let N denote the set of all natural numbers. Define two binary relations on N as $R_1 = \{(x, y) \in N \times N : 2x + y = 10\}$ and $R_2 = \{(x, y) \in N \times N : x + 2y = 10\}$. Then

(1) Both R_1 and R_2 are transitive relations

(2) Range of R_2 is {1, 2, 3, 4}.

(3) Range of R_1 is {2, 4, 8}

(4) Both R_1 and R_2 are symmetric relations.

Ans. **(2)**

Sol. $R_1 = \{(1, 8), (2, 6), (3, 4), (9, 2)\}$

$R_2 = \{(8, 1), (6, 2), (4, 3), (2, 4)\}$

Range of $R_2 = \{1, 2, 3, 4\}$

3. The coefficient of x^2 in the expansion of the product $(2- x^2)\cdot((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$ is :

(1) 107 (2) 108 (3) 155 (4*) 106

Ans. **(4)**

Sol. coefficient of x^2 = 2 coefficient of x^2 in $((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$ – constant term

$$(1 + 2x + 3x^2)^6 = \sum_{r=0}^{6} {}^6c_r(2x+3x^2)^r$$

$$= {}^6C_0 + {}^6C_1(2x + 3x^2) + {}^6C_2(2x + 3x^2)^2 + \ldots.$$

coefficient of $x^2 = 2(18 + 60 - 24) - 2$

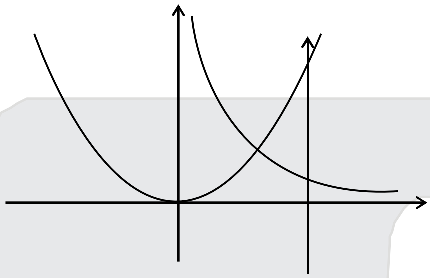
$= 108 - 2 = 106$

4. If the area of the region bounded by the curves, $y = x^2$, $y = \frac{1}{x}$ and the lines y = 0 and x = t (t > 1) is 1 sq. unit, then t is equal to :

(1*) $e^{\frac{2}{3}}$ (2) $e^{\frac{3}{2}}$ (3) $\frac{3}{2}$ (4) $\frac{4}{3}$

Ans. **(1)**

Sol. $\int_x^1 x^2 dx + \int + \frac{1}{x} dx = 1$



$\frac{1}{3} + \ell nt = 1 \quad \Rightarrow \quad t = e^{\frac{2}{3}}$

5. If the length of the latus rectum of an ellipse is 4 units and the distance between a focus and its nearest vertex on the major axis is $\frac{3}{2}$ units, then its eccentricity is :

(1) $\frac{2}{3}$ (2) $\frac{1}{2}$ (3) $\frac{1}{9}$ (4) $\frac{1}{3}$

Ans. **(4)**

Sol. $\frac{2b^2}{a} = 4 \Rightarrow \quad b^2 = 2a$

$b^2 = a^2 (1 - e^2)$, $a (1 - e) = \frac{3}{2}$

$2 = a(1 - e) (1 + e)$

$2 = \frac{3}{2}(1 + e) \quad e = \frac{1}{3}$

6. The number of numbers between 2,000 and 5,000 that can be formed with the digits 0, 1, 2, 3, 4 (repetition of digits is not allowed) and are multiple of 3 is :

(1) 36 (2) 30 (3) 24 (4) 48

Ans. **(2)**

Sol. number can be formed y (0, 1, 2, 3) or (0, 2, 3, 4)
number of 4 digits number = 2 × 3 ! + 3 × 3 ! = 30

7. Two different families A and B are blessed with equal number of children. There are 3 tickests to be distributed amongst the children of these families so that no child gets more than one ticket. If the probability that all the tickets go to the children of the family B is $\frac{1}{12}$, then the number of children in each family is :

(1) 6 (2) 5 (3) 3 (4) 4

Ans. **(2)**

Sol. Let n number of children are there in each family

$\frac{1}{12} = \frac{{}^nC_3.3!}{{}^{2n}C_3.3!}$

$\frac{{}^nC_3}{{}^{2n}C_3} = \frac{1}{12} \quad n = 5$

8. $\lim\limits_{x\to 0}\dfrac{(27+x)^{\frac{1}{3}}-3}{9-(27+x)^{\frac{2}{3}}}$ equals :

(1) $-\frac{1}{6}$ (2) $\frac{1}{6}$ (3) $\frac{1}{3}$ (4) $-\frac{1}{3}$

Ans. **(1)**

Sol. $\lim\limits_{x\to 0}\dfrac{3\left[\left(1+\frac{x}{27}\right)^{\frac{1}{3}}-1\right]}{9\left[1-\left(1+\frac{x}{27}\right)^{\frac{2}{3}}\right]}$

$\lim\limits_{x\to 0}\dfrac{1}{3}\left[\dfrac{\frac{x}{81}}{-\frac{2}{3}\cdot\frac{x}{27}}\right]=\dfrac{-1}{6}$

9. Let p, q and r be real numbers ($p \neq q$, $r \neq 0$), such that the roots of the equation $\dfrac{1}{x+p}+\dfrac{1}{x+q}=\dfrac{1}{r}$ are equal in magnitude but opposite in sign, then the sum of squares of these roots is equal to :

(1) $p^2 + q^2$ (2) $\dfrac{p^2+q^2}{2}$ (3) $2(p^2 + q^2)$ (4) $p^2 + q^2 + r^2$

Ans. **(1)**

Sol. $(2x + p + q)\, r = (x + p)(x + q)$

$x^2 + (p + q - 2r)\, x + pq - pr - qr = 0$

$p + q = 2r$ ....(i)

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 0 - 2\,[pq - pr - qr] = -2pq + 2r\,(p + q) = -2pq + (p + q)^2 = p^2 + q^2$

10. Let $\dfrac{1}{x_1}, \dfrac{1}{x_2}, \ldots, \dfrac{1}{x_n}$ ($x_i \neq 0$ for i = 1, 2, ..., n) be in A.P. such that $x_1 = 4$ and $x_{21} = 20$. If n is the least positive integer for which $x_n > 50$, then $\sum\limits_{i=1}^{n}\left(\dfrac{1}{x_i}\right)$ is equal to :

(1) 3 (2) $\frac{1}{8}$ (3) $\frac{13}{4}$ (4) $\frac{13}{8}$

Ans. **(3)**

Sol. $\dfrac{1}{4} + 20.d = \dfrac{1}{20}$

$d = \dfrac{-1}{100}$

$\dfrac{1}{x_n} < \dfrac{1}{50}$

$\dfrac{1}{4} - \dfrac{n-1}{100} < \dfrac{1}{50} \quad \Rightarrow \quad n > 24$

$n = 25$

$\sum\limits_{i=1}^{25}\left(\dfrac{1}{x_i}\right) = \dfrac{25}{2}\left[2\times\dfrac{1}{4}-\dfrac{1}{100}\times 24\right] = \dfrac{13}{4}$

11. The differential equation representing the family of ellipses having foci either on the x-axis or on the y-axis, centre at the origin and passing through the point (0, 3) is :

(1*) $xy\,y' - y^2 + 9 = 0$ (2) $xy\,y'' + x(y')^2 - y\,y' = 0$

(3) $xy\,y' + y^2 - 9 = 0$ (4) $x + y\,y'' = 0$

Ans. **(1)**

Sol. Equation of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Passes (0, 3) $\Rightarrow$ $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ ....(1) $\frac{x}{a^2} = -\frac{y}{9}\frac{dy}{dx}$

$\frac{2x}{a^2} + \frac{2y}{9}\frac{dy}{dx} = 0$ ....(2) $\frac{1}{a^2} = -\frac{y}{9x}y^1$

By (1) & (2) D. equation is

$$-\frac{xy}{9}y^1 + \frac{y^2}{9} = 1$$

$\Rightarrow$ $xy\,y^1 - y^2 + 9 = 0$

12. The sum of the intercepts on the coordinate axes of the plane passing trhough the point (–2, –2, 2) and containing the line joining the points (1, –1, 2) and (1, 1, 1), is :

(1) 4 (2) 12 (3) –8 (4) –4

Ans. **(4)**

Sol. Equation plane

$$\begin{vmatrix} x+2 & y+2 & z-2 \\ -3 & -1 & 0 \\ -3 & -3 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $-(x + 2) + 3(y + 2) + 6(z - 2) = 0$

$\Rightarrow$ $x - 3y - 6z + 8 = 0$

sum of intercepts $= -8 + \frac{8}{3} + \frac{8}{6} = -4$

13. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^{20}$. Then the sum of the elements of the first column of B is :

(1) 210 (2) 211 (3) 251 (4) 231

Ans. **(4)**

Sol. $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$; $A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 10 & 4 & 1 \end{bmatrix}$ ..... $A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 20 & 1 & 0 \\ 210 & 20 & 1 \end{bmatrix}$

Sum of the elements of first column = 231

14. Let A, B and C be three events, which are pair-wise independent and $\bar{E}$ denotes the complement of an event E. If $P(A \cap B \cap C) = 0$ and $P(C) > 0$, then $P[(\bar{A} \cap \bar{B})| C]$ is equal to :

(1) $P(\bar{A}) - P(B)$ (2) $P(\bar{A}) - P(\bar{B})$ (3) $P(\bar{A}) + P(\bar{B})$ (4) $P(A) + P(\bar{B})$

Ans. **(1)**

Sol. $P[(\bar{A} \cap \bar{B})|C] = \dfrac{P[(\bar{A} \cup \bar{B}) \cap C]}{P(C)}$

$= \dfrac{P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}{P(C)}$

$= \dfrac{P(C) - P(A)P(C) + P(B)P(C)}{P(C)}$

$= 1 - P(A) - P(B)$

$= P\left(\bar{A}\right) - P(B)$ or $P\left(\bar{B}\right) - P(A)$

15. If $p \to (\sim p \vee \sim q)$ is false, then the truth values of p and q are respectively :

(1) F, F (2) T, T (3) F, T (4) T, F

Ans. **(2)**

Sol. $P \to (\sim p \vee \sim q)$

p	q	$\sim p \vee \sim q$	$p \to (\sim p \vee \sim q)$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

16. If the function f defined as $f(x) = \dfrac{1}{x} - \dfrac{k-1}{e^{2x}-1}$, $x \neq 0$, is continuous at x = 0, then the ordered pair (k, f(0)) is equal to :

(1) (2, 1) (2) (3, 1) (3) (3, 2) (4) $\left(\dfrac{1}{3}, 2\right)$

Ans. **(2)**

Sol. $f(x) = \dfrac{1}{x} - \dfrac{k-1}{e^{2x}-1}$; $x \neq 0$

f(x) is continuous at x = 0

$\Rightarrow f(0) = \lim\limits_{x \to 0} \dfrac{1}{x} - \dfrac{k-1}{e^{2x}-1}$

$= \lim\limits_{x \to 0} \dfrac{(1 + (2x) + \frac{1}{2!}(2x)^2 + ...(-1 - x(k-1)}{2x^2\left(\dfrac{e^{2x}-1}{2x}\right)}$

Clearly k = 3 and f(0) = 1

17. If the angle between the lines, $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{5-x}{-2} = \frac{7y-14}{p} = \frac{z-3}{4}$ is $\cos^{-1}\left(\frac{2}{3}\right)$, then p is equal to :

(1) $\frac{2}{7}$ (2) $\frac{7}{2}$ (3) $-\frac{4}{7}$ (4) $-\frac{7}{4}$

Ans. **(2)**

Sol. $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x-5}{2} = \frac{y-2}{P/7} = \frac{z-3}{4}$

Angle between both lines is $\cos^{-1}\left(\frac{2}{3}\right) = \cos^{-1}\left(\frac{4+\frac{2P}{7}+4}{3.\sqrt{4+\frac{P^2}{49}+16}}\right)$

$\Rightarrow \frac{2}{3} = \frac{56+2P}{3\sqrt{P^2+980}} \Rightarrow \sqrt{P^2+980} = P + 28 \Rightarrow P^2 + 980 = P^2 + 56P + 784 \Rightarrow 56P = 196 \Rightarrow P = \frac{7}{2}$

18. The locus of the point of intersection of the lines, $\sqrt{2}\,x - y + 4\sqrt{2}\,k = 0$ and $\sqrt{2}\,kx + ky - 4\sqrt{2} = 0$ (k is any non-zero real parameter), is :

(1) an ellipse whose eccentricity is $\frac{1}{\sqrt{3}}$.

(2) a hyperbola whose eccentricity is $\sqrt{3}$

(3) a hyperbola with length of its transverse axis $8\sqrt{2}$

(4) an ellipse with length of its major axis $8\sqrt{2}$.

Ans. **(3)**

Sol. $\sqrt{2}\,x - y + 4\sqrt{2}\,k = 0$ ....(i)

$\sqrt{2}\,kx + ky - 4\sqrt{2} = 0$ ....(ii)

Eliminating k by (i) and (ii)

$\left(\sqrt{2}x+y\right)\left(\frac{\sqrt{2}x-y}{-4\sqrt{2}}\right) = 4\sqrt{2}$

$2x^2 - y^2 = -32$

$\frac{y^2}{32} - \frac{x^2}{16} = 1$ Hyperbola

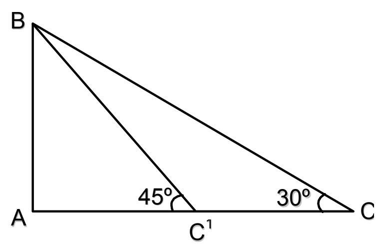
$e = \sqrt{1+\frac{16}{32}} = \sqrt{\frac{3}{2}}$ and length of transverse axis = $8\sqrt{2}$

19. A man on the top of a vertical tower observes a car moving at a uniform speed towards the tower on a horizonatal road. If it takes 18 min. for the angle of depression of the car to change from 30° to 45° ; then after this, the time taken (in min.) by the car to reach the foot of the tower, is :

(1) $\frac{9}{2}(\sqrt{3}-1)$ (2) $18(1+\sqrt{3})$ (3) $18(\sqrt{3}-1)$ (4) $9(1+\sqrt{3})$

Ans. **(4)**

Sol. Let length of tower = h



$\Rightarrow$ AC' = AB = h

and AC = AB cot 30º = $\sqrt{3}$ h $\Rightarrow$ CC' = $(\sqrt{3}-1)$ h

Time taken by car form C to C' = 18 min

$\Rightarrow$ time take by car to reach the foot of the tower = $\frac{18}{\sqrt{3}-1}$ min.

= $9(\sqrt{3}+1)$ min

20. If an angle A of a ΔABC satisfies $5\cos A + 3 = 0$, then the roots of the quadratic equaiton, $9x^2 + 27x + 20 = 0$ are :

(1) sec A, cotA (2) secA, tanA (3) tanA, cosA (4) sinA, secA

Ans. **(2)**

Sol. $5\cos A + 3 = 0 \Rightarrow \cos A = -\frac{3}{5}$ clearly $A \in (90^\circ, 180^\circ)$

Now roots of equation $9x^2 + 27x + 20 = 0$ are $-\frac{5}{3}$ and $-\frac{4}{3}$

$\Rightarrow$ Roots secA and tanA

21. If a circle C, whose radius is 3, touches externally the circle, $x^2 + y^2 + 2x - 4y - 4 = 0$ at the point (2, 2), then the length of the intercept cut by this circle C, on the x-axis is equal to :

(1) $2\sqrt{3}$ (2) $\sqrt{5}$ (3) $3\sqrt{2}$ (4) $2\sqrt{5}$

Ans. **(4)**

Sol. Centre of given circle = (–1, 2)

and radius = $\sqrt{1+4+4} = 3$

centre of required circle (5,2)

or (–4,2)

length of intercept on x-axis will be square in both circle

so one required circle $(x-5)^2 + (y-2)^2 = 3^2$

$x^2 + y^2 - 10x - 4y + 20 = 0$

Length of x intercept = $2\sqrt{g^2 - c}$

$= 2\sqrt{25-20} = 2\sqrt{5}$

22. Let P be a point on the parabola, $x^2 = 4y$. If the distance of P from the centre of the circle, $x^2 + y^2 + 6x + 8 = 0$ is minimum, then the equation of the tangent to the parabola at P, is :

(1) $x + y + 1 = 0$ (2) $x + 4y - 2 = 0$ (3) $x + 2y = 0$ (4) $x - y + 3 = 0$

Ans. **1**

Sol. Let P $(2t, t^2)$

equation normal at P to $x^2 = 4y$ be

$y - t^2 = -\frac{1}{t}(x - 2t)$

it passes through (–3,0)

$0 - t^2 = -\frac{1}{t}(-3 - 2t)$

$t^3 + 2t + 3 = 0$

$(t + 1)(t^2 - t + 3) = 0$

$\Rightarrow t = -1$

Point P is (–2,1)

equation of tangent to $x^2 = 4y$ at (–2,1)

$x(-2) = 2(y + 1)$

$x + y + 1 = 0$

23. If $f(x) = \int_0^x t(\sin x - \sin t)\,dt$ then :

(1) $f'''(x) - f''(x) = \cos x - 2x\sin x$
(2) $f'''(x) + f''(x) - f'(x) = \cos x$
(3) $f'''(x) + f''(x) = \sin x$
(4) $f'''(x) + f'(x) = \cos x - 2x\sin x$

Ans. 4

Sol. $f(x) = \int_0^x t(\sin x - \sin t)dt$

$f(x) = \sin x \int_0^x t\,dt - \int_0^x t\sin t\,dt$

$f'(x) = (\sin x)\,x + \cos x \int_0^x t\,dt - x\sin x$

$f'(x) = \cos x \int_0^x t\,dt$

$f''(x) = (\cos x)\,x - (\sin x)\int_0^x t\,dt$

$f'''(x) = x(-\sin x) + \cos x - (\sin x)x - (\cos x)\int_0^x t\,dt$

$f'''(x) + f'(x) = \cos x - 2x\sin x$

24. The number of values of k for which the system of linear equations,
$(k + 2)x + 10y = k$
$kx + (k + 3)y = k - 1$ has no soution, is :

(1) 1
(2) 2
(3) 3
(4) 4

Ans. **(1)**

Sol. For no solution

$\frac{k+2}{k} = \frac{10}{k+3} \neq \frac{k}{k-1}$

$(k + 2)(k + 3) = 10k$

$k^2 - 5k + 6 = 0 \Rightarrow k = 2,3$

$k \neq 2$ for $k = 2$ both lines identical

so $k = 3$ only

so number of values of k is 1

25. If $\int \frac{\tan x}{1+\tan x+\tan^2 x}dx = x - \frac{K}{\sqrt{A}}\tan^{-1}\left(\frac{K\tan x+1}{\sqrt{A}}\right) + C$, (C is a constant of integration), then the ordered pair (K, A) is equal to

(1) (2, 1)
(2) (2, 3)
(3) (–2, 1)
(4) (–2, 3)

Ans. **(2)**

Sol. $I = \int \frac{\tan}{1+\tan x+\tan^2 x}dx$

$\int \left(1 - \frac{\sec^2 x}{1+\tan x+\tan^2 x}\right)dx$

$= x - \int \frac{dt}{1+t+t^2}$, where $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$= \int \frac{dt}{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}} = x - \frac{1}{\sqrt{3}/2}\tan^{-1}\left(\frac{t+\frac{1}{2}}{\sqrt{3}/2}\right) + C = x - \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x+1}{\sqrt{\sqrt{3}}}\right) + C$

$= k = 2$, $A = 3$.

26. The least positive integer n for which $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n = 1$, is

(1) 2 (2) 5 (3) 6 (4) 3

Ans. **(4)**

Sol. $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n = 1$

$\left(\frac{-2\omega^2}{-2\omega}\right)^n = 1$

$\omega^n = 1$

least positive integer value of n is 3.

27. The sum of the first 20 terms of the series $1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \frac{31}{16} + \ldots$, is :

(1) $39 + \frac{1}{2^{19}}$ (2) $38 + \frac{1}{2^{20}}$ (3) $38 + \frac{1}{2^{19}}$ (4) $39 + \frac{1}{2^{20}}$

Ans. **(3)**

Sol. $1+\frac{3}{2}+\frac{7}{4}+\frac{15}{8}+\frac{31}{16}+\ldots\ldots$

$= (2-1) + \left(2-\frac{1}{2}\right)+\left(2-\frac{1}{4}\right)+\left(2-\frac{1}{8}\right)+\left(2-\frac{1}{16}\right)+\ldots\ldots\ldots\text{upto 20 terms}$

$= 40 - \left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\text{up to 20 terms}\right)$

$= 40 - \left(\frac{1-\left(\frac{1}{2}\right)^{20}}{1-\frac{1}{2}}\right) = 40 - 2 + \frac{1}{2^{19}} = 38 + \frac{1}{2^{19}}$

28. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{j} - \hat{k}$ and a vector $\vec{b}$ be such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 3$. Then $|\vec{b}|$ equals :

(1) $\frac{11}{3}$ (2) $\frac{11}{\sqrt{3}}$ (3) $\sqrt{\frac{11}{3}}$ (4) $\frac{\sqrt{11}}{3}$

Ans. **(3)**

Sol. $\vec{a}\times\vec{b} = \vec{c}$

$\vec{a}\times(\vec{a}\times\vec{b}) = \vec{a}\times\vec{c}$

$(\vec{a}.\vec{b})\vec{a} - (\vec{a}.\vec{a})\vec{b} = \vec{a}\times\vec{c}$

$3\vec{a} - 3\vec{b} = -2\hat{i}+\hat{j}+\hat{k}$

$3i + 3j + 3\hat{k} - 3\hat{b} = -2\hat{i} + \hat{j} + \hat{k}$

$\vec{b} = \frac{1}{3}(5\hat{i}+2\hat{j}+2\hat{k})$

$|\vec{b}| = \frac{\sqrt{25+4+4}}{3}$

$|\vec{b}| = \sqrt{\frac{11}{3}}$

29. The mean and the standard deviation (s.d.) of five observations are 9 and 0, respectively. If one of the observations is changed such that the mean of the new set of five observations becomes 10, then their s.d. is :

(1) 0 (2) 2 (3) 4 (4) 1

Ans. **(1)**

Sol. Standard deviations with be same

so S.D is 0

30. Let M and m be respectively the absolute maximum and the absolute minimum values of the function, $f(x) = 2x^3 - 9x^2 + 12x + 5$ in the interval [0, 3]. Then M – m is equal to :

(1) 9 (2) 4 (3) 1 (4) 5

Ans. **(1)**

Sol. $f(x) = 2x^3 - 9x^2 + 12 + 5, \quad x \in [0, 3]$

$f'(x) = 6x^2 - 18x + 12$

$f'(x) = 6(x-1)(x-2)$

$f(1) = 10$

$f(2) = 9$

$f(0) = 5$

$f(3) = 14$

$M = 14, \qquad m - 5 \quad \Rightarrow \qquad M - m = 9$

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